On Optimizing Non-Asymptotic Throughput of Wireless Mesh Networks

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Abstract—In this paper, throughput performance of WMNs is studied. In our model, a regular grid backbone network overlays on a random ad hoc network. We propose a framework to calculate non-asymptotic throughput, which can be obtained by computing several deterministic parameters. Two problems are investigated. In Maximum Throughput Partition (MTP) problem, the ideal throughput is achieved by optimally partitioning the network with a proper number of backbone nodes. In Maximum Throughput Partition with Hops' number Constraint (MTPHC) problem, a similar problem is studied but with constraint on the average number of hops in the backbone network. The results show that it is critical to find an appropriate size of the backbone network for a WMN, especially when the hops' number constraint is imposed. Our solution of MTPHC problem can be also used to obtain the ideal transmission range when less-than-optimal number of backbone nodes is deployed. Comparing with the minimum transmission range, the ideal one can achieve the same optimal throughput but effectively reduce the average number of hops in mesh backbone communications.

Keywords—wireless mesh networks; non-asymptotic throughput; backbone optimization

I. INTRODUCTION

In a wireless mesh network (WMN), a traditional ad hoc network is overlaid with an infrastructure network called mesh backbone. Mesh backbone comprises wireless mesh routers, which are powerful devices without constraints of energy, computing power, and memory. Usually they are distributed in a static and deterministic manner. WMNs offer all the advantages of ad hoc wireless networks plus many extra benefits from the infrastructure architecture. Wireless mesh backbone can be rapidly deployed with minimal cost and provides a robust, efficient, reliable, and flexible system that supports the network access for mesh clients. Mesh backbone can also provide mesh clients with various services and resources through their gateway and bridging functions. With infrastructure support, the complexity of communication protocols in mesh clients can be reduced significantly. All these advantages reinforce WMNs as a promising wireless technology for numerous applications, e.g. broadband home networking, community and enterprise networking, public Internet access, and so on.

Many research problems still remain open in WMNs [1]. Among them, one of the most challenging research topics is to study the throughput of WMNs. Throughput capacity of multi-hop wireless networks has been studied by many recent works.

Gupta and Kumar [2] derived the per-node throughput capacity for static ad hoc networks. The throughput capacity of mobile ad hoc networks was analyzed by Grossglauser and Tse [3]. The capacity of hybrid ad hoc networks was investigated in [4, 5, 6]. And the theoretical capacity of infrastructure WMNs was recently derived in [7]. All the above works assumed 100% multiple access efficiency so that no practical MAC protocols have been considered. Silvester and Kleinrock [8] incorporated slotted ALOHA MAC protocol into their capacity analysis of multi-hop networks with regular structure. By analyzing the same MAC protocol, Liu and Haenggi [9] achieved the throughput of fading sensor networks with both regular and random topologies.

In most of above research, theoretical results have been obtained as asymptotic value by assuming that the size of the network goes to infinity. Since real networks always have limited size, these asymptotic results provide very little information for practical network design. In this paper, we propose a framework within which non-asymptotic throughput of WMNs can be obtained by computing several deterministic parameters. While node density was proved to be a critical factor for throughput performance of conventional ad hoc networks [10, 11], we demonstrate in this paper that under various conditions and constraints, ideal throughput of WMNs can be achieved by deploying a proper number of backbone nodes.

The rest of this paper is organized as follows. In Section II, a typical WMN model is described. The Maximum Throughput Partition (MTP) Problem and the Maximum Throughput Partition with Hops' number Constraint (MTPHC) Problem are formulated in Section III. MTP and MTPHC solutions are presented in Sections IV and V, respectively. The numeric results are obtained by simulations in Section VI. This paper is concluded in Section VII.

II. SYSTEM MODEL

In a typical WMN, as shown in Fig. 1, n mesh clients are assumed to be uniformly and independently distributed on a square . R is partitioned evenly into small cells , and a mesh router is placed in the center of each cell. Let m denote the number of mesh routers, then . In what as follows, we will limit the case of interests to that where . Mesh routers
constitute a wireless mesh backbone providing a wireless infrastructure for mesh clients. In each cell, mesh clients are connected to the mesh router like a star topology, i.e., no direct communication is available among mesh clients, and the mesh router works as a hub for mesh clients. Such a WMN is referred as an infrastructure WMN in [1], which will be very popular in future WMN applications.

Each mesh client is a data source, which randomly chooses another mesh client as its destination. The cells where the source client and the destination client are located are defined as the source cell and the destination cell, respectively. Unlike mesh clients, mesh routers are neither data source nor data destination; they only route and forward data for mesh clients. Three types of communications are identified in the network: 1) Uplink Communications: a source client transmits data to its mesh router in the source cell; 2) Inter-cell Communications: mesh routers relay data in a multi-hop fashion from the source cell to the destination cell on the mesh backbone; 3) Downlink Communications: the mesh router in the destination cell transmits data to the destination client. Both uplink and downlink communications are also defined as intra-cell communications.

Each mesh router is equipped with two radio interfaces such that it transmits at $W_1$ bits/s in inter-cell communications and it transmits at $W_2$ bits/s in downlink communications. Each mesh client transmits at $W_2$ bits/s in uplink communications. We assume that $W_1$ and $W_2$ are orthogonal. Note that mesh routers and mesh clients use the same radio interface in intra-cell communications. And we split the bandwidth evenly for uplink and downlink communications, respectively so that all three types of communications don’t interfere with each other. Since all uplink traffic will finally go to downlink in this model, such a bandwidth-split scheme is reasonable and it can be implemented easily by applying a simple TDMA scheme. In addition, mesh routers can receive packets from only one sender at a time and cannot transmit and receive packets simultaneously. The same constraint is imposed on mesh clients.

We assume that a transmission is successful if the signal-to-noise-and-interference ratio (SINR) is above a certain threshold $\Theta$. The SINR is given by $P_d d^{-\alpha} / (N_0 + I)$, where $P_d$ denotes the transmit power, $d$ is the distance between the transmitter and the receiver, $\alpha$ is the path loss exponent, $N_0$ denotes the noise power, and $I$ is the interference power, which is the sum of the received power from all the undesired transmitters. In what as follows, we assume that $N_0 = 0$, i.e., in wireless networks, dominant interference is from other nodes instead of background noise.

We use the slotted ALOHA as the multi-access protocol and a shortest path routing protocol is used in inter-cell communications.

III. PROBLEM FORMULATION

In the above WMN model, given $n$, $W_1$, $W_2$ and specific transmission, scheduling and routing protocols, per client throughput, denoted as $TP(m, r)$, is a function of the number of mesh routers $m$ and the common transmission range all mesh routers choose in inter-cell communications, denoted as $r$. Since $W_1$ and $W_2$ are orthogonal, $TP(m, r)$ can be obtained by computing $TP_{m_1}(m, r)$ and $TP_{m_2}(m)$ separately, where $TP_{m_1}(m, r)$ is feasible per client throughput in inter-cell communications and $TP_{m_2}(m)$ is feasible per client throughput in intra-cell communications. Obviously,

$$TP(m, r) = \min \{TP_{m_1}(m, r), TP_{m_2}(m)\}.$$ 

We now give a precise formulation for the two problems that will be addressed in this paper: (i) The Maximum Throughput Partition (MTP) Problem and (ii) The Maximum Throughput Partition with Hops’ number Constraint (MTPHC) Problem. In both the problems, we partition the network such that the worst case of per client throughput is maximized, which is denoted as $\min TP(m, r)$. The worst case means that such throughput is obtained by considering the most heavily loaded nodes in the network. For example, in intra-cell communications, throughput is always calculated in the cell with the largest number of mesh clients and in inter-cell communications, throughput is always obtained on the routers in the center of the backbone network. Hence, the obtained throughput is a guaranteed per client throughput that can be achieved by every node in the WMN.

**Problem MTP**: In the WMN model, given $n$, $W_1$, $W_2$ and specific transmission, scheduling and routing protocols, the network region $R$ is partitioned into $m$ small cells such that,

$$\min TP(m, r) = \min \{\min TP_{m_1}(m, r), \min TP_{m_2}(m)\},$$

$$m = i^2 \text{ and } 1 < i < \sqrt{n},$$

is maximized.

**Problem MTPHC**: In the above WMN model, given $n$, $W_1$, $W_2$ and specific transmission, scheduling and routing protocols, the network region $R$ is partitioned into $m$ small cells such that,

$$\min TP(m, r) = \min \{\min TP_{m_1}(m, r) | \bar{h} \leq c, \min TP_{m_2}(m)\},$$

$$m = i^2 \text{ and } 1 < i < \sqrt{n},$$

is maximized, where $\bar{h}$ denotes the average number of hops per bits in inter-cell communications and $c$ is a constant.
IV. MTP Solution

Proposition 1: Given \( n \), \( W_1 \) and specific transmission, scheduling and routing protocols, the worst case of feasible per client throughput in inter-cell communications can be obtained as follows:

\[
\min TP_{\text{m}}(m, r) = \frac{W_i E_i(m, r)}{h(m, r) \overline{N}_{\text{max}}(n, m)}, m = \overline{i} \text{ and } 1 < \overline{i} \sqrt{n},
\]

where \( E_i(m, r) \) denotes resource sharing efficiency of inter-cell communications, \( h(m, r) \) denotes average number of hops per bit in inter-cell communications, \( \overline{N}_{\text{max}}(n, m) \) denotes the expected maximal number of mesh clients in any cell.

Proposition 2: Given \( n \), \( W_2 \) and specific transmission and scheduling protocols, the worst case of feasible per client throughput in intra-cell communications can be obtained as follows:

\[
\min TP_{\text{i}}(m) = \frac{W_i E_i(n, m)}{2 \overline{N}_{\text{max}}(n, m)}, m = \overline{i} \text{ and } 1 < \overline{i} \sqrt{n},
\]

where \( E_i(n, m) \) denotes resource sharing efficiency of intra-cell communications.

Corollary 1: Given \( n \), \( W_1 \), \( W_2 \) and specific transmission, scheduling and routing protocols, the worst case of feasible per client throughput of WMNs can be obtained as follows:

\[
\min TP(m, r) = \min \left\{ \frac{W_i E_i(m, r)}{h(m, r) \overline{N}_{\text{max}}(n, m)}, \frac{W_i E_i(n, m)}{2 \overline{N}_{\text{max}}(n, m)} \right\},
\]

\[
m = \overline{i} \text{ and } 1 < \overline{i} \sqrt{n}.
\]

In what as follows we will give the solution of computing \( E_i(m, r), E_i(n, m), \overline{h}(m, r), \) and \( \overline{N}_{\text{max}}(n, m) \).

A. Sharing efficiency of inter-cell communications

The sharing efficiency here is equivalent to the multiple access efficiency of slotted ALOHA protocol in the regular grid backbone network. A similar case has been well studied in [8], in which one can find the detailed proof of proposition 3.

Proposition 3: In the slotted ALOHA backbone network, assuming that nodes transmit at equal power levels with probability \( p \), the success probability of a transmission given a desired transmitter-receiver distance \( r \) and \( (m-2) \) other nodes at distances \( d_i \) \( (i = 1, \ldots, m-2) \) is

\[
P_{\text{s}}(m, r, p) = \prod_{i=1}^{m-2} \left( 1 - \frac{\overline{\Theta} p}{(d_i/r)\overline{\Theta} + \overline{\Theta}} \right),
\]

In the above formula, \( d_i \) is easy to get since all mesh routers are placed regularly in the backbone network. Recall that \( E_i(m, r) \) will be obtained by assuming the receiver is located at the center of the backbone network.

Therefore, \( E_i(m, r, p) \) is given by

\[
E_i(m, r, p) = p(1-p)P_{\text{s}}(m, r, p),
\]

since \( p \) is the probability that a source router transmits and \( 1-p \) is the probability that its 1-hop receiver does not transmit in the same timeslot.

Finally, \( E_i(m, r) \) can be obtained by

\[
E_i(m, r) = \max_{p = p_{\text{opt}}} \{ E_i(m, r, p) \},
\]

i.e., we can always obtain the optimal transmission probability.

Fig. 2 and Table 1 give examples of \( E_i(m, r) \) from simulations. In the examples, the minimum transmission range is adopted in inter-cell communications, i.e., \( r = l / \sqrt{m} \).

<table>
<thead>
<tr>
<th>( m )</th>
<th>4</th>
<th>9</th>
<th>16</th>
<th>25</th>
<th>36</th>
<th>49</th>
<th>64</th>
<th>81</th>
<th>100</th>
<th>121</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_i )</td>
<td>0.154</td>
<td>0.079</td>
<td>0.071</td>
<td>0.064</td>
<td>0.062</td>
<td>0.060</td>
<td>0.060</td>
<td>0.059</td>
<td>0.058</td>
<td>0.058</td>
</tr>
<tr>
<td>( m )</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
<td>256</td>
<td>289</td>
<td>324</td>
<td>361</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>( E_i )</td>
<td>0.058</td>
<td>0.058</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
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</tbody>
</table>

B. Sharing efficiency of intra-cell communications

In uplink communications, transmissions are scheduled by a TDMA-slotted ALOHA combined scheme. As shown in Fig. 3, first, a TDMA scheduling scheme guarantees that each cell can get a time slot for transmission every 4 time slots such that there is no transmission occurring simultaneously in its 8 neighboring cells. Second, in the specific time slot, all nodes in the cell content for the uplink communications using slotted ALOHA. Therefore,

\[
E_i(n, m) = \left( 1 - \frac{1}{\overline{N}_{\text{max}}} \right)^{\overline{N}_{\text{max}} - 1} \approx \frac{1}{4e}
\]

C. Average number of hops

The average number of hops can be obtained by
where $h_j(m,r)$ denotes the number of hops from node $i$ to node $j$ using shortest path routing. Table 2 displays examples of $\bar{h}(m,r)$ from simulations.

Table 2: Comparison of $\bar{h}(m, r = l/\sqrt{m})$ and $\bar{h}(m, r = \sqrt{2l}/\sqrt{m})$

<table>
<thead>
<tr>
<th>$m$</th>
<th>16</th>
<th>36</th>
<th>64</th>
<th>100</th>
<th>144</th>
<th>196</th>
<th>256</th>
<th>324</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{h}(m, r = l/\sqrt{m})$</td>
<td>2.67</td>
<td>4.00</td>
<td>5.33</td>
<td>6.67</td>
<td>8.00</td>
<td>9.33</td>
<td>10.67</td>
<td>12.00</td>
<td>13.33</td>
</tr>
<tr>
<td>$\bar{h}(m, r = \sqrt{2l}/\sqrt{m})$</td>
<td>1.90</td>
<td>2.82</td>
<td>3.75</td>
<td>4.68</td>
<td>5.61</td>
<td>6.54</td>
<td>7.48</td>
<td>8.41</td>
<td>9.34</td>
</tr>
</tbody>
</table>

D. Expected maximum number of mesh clients in any cell

Let $N_j(n,m)$ be the random variable denoting the number of mesh clients falling into the $j$-th cell. The expected maximum number of mesh clients in any cell $\bar{N}_{\text{max}}(n,m)$ is defined as follows:

$$\bar{N}_{\text{max}}(n,m) = E[\max_j(N_j(n,m))], 1 \leq j \leq m.$$ 

$\bar{N}_{\text{max}}(n,m)$ is used to compute throughput in the most heavily loaded cell. It is different from the mean number of client in each cell, which is $n/m$. Fig.4 displays both the numeric results of $\bar{N}_{\text{max}}(n,m)$ and $n/m$ by simulations when there are 400 mesh clients in the network.

V. MTPHC SOLUTION

In the next section, our simulation results will show that for MTP Problem, optimal throughput can always be achieved by using the shortest transmission range in inter-cell communications. The shortest transmission range is the minimal transmission range that keeps the backbone network fully connected. However, in order to meet the latency requirement, sometimes longer transmission range must be adopted to reduce the number of hops, which results in much lower sharing efficiency of inter-cell communications. Therefore, lower throughput is expected. In such cases, optimal network partition will be very different from the cases without hops’ number constraint.

MTPHC problem can be solved by computing MTP results with a specific transmission range, which makes the average number of hops in inter-cell communications satisfy the requirement, i.e. $\bar{h}(m,r) \leq c$. The algorithm is as follows.

**Algorithm MTPHC Framework**

1: for all $m$
2: Choose the minimum transmission range for inter-cell communications, i.e. $r = l/\sqrt{m}$.
3: if $\bar{h}(m,r) \leq c$ then calculate the MTP results indicated by (1) and (4).
4: else increase $r$ to the next level and go back to step 3.
5: return the optimal partition number and throughput.

VI. SIMULATION RESULTS

In this section, we calculate the numeric results by simulations. For both problems, we assume $W_1 = 54Mbits/s$, $W_2 = 11Mbits/s$, $\Theta = 2$, $\alpha = 4$, and $l = 1000m$.

For MTP problem, Fig.5 shows the per client throughput by varying the number of mesh routers. For each $n$, throughput of WMN is decided by 2 curves. Here flat curves indicate feasible per client throughput in inter-cell communications while steep ones indicate feasible per client throughput in intra-cell communications. They are obtained by (2) and (3), respectively. As demonstrated in (4), per client throughput of the WMN is thus achieved by always taking the smaller one of the two throughputs. With 300 randomly distributed mesh clients, the optimal worst case of per client throughput has been achieved as 58.1Kbits/s by partitioning the network into 10by10 grid, i.e. deploying 100 mesh routers. Similarly, in the case of $n = 400$ and $n = 500$, the optimal throughput is 48.5Kbits/s and 42.2Kbits/s by deploying 144 and 196 mesh routers, respectively. Therefore, more mesh clients need the backbone comprising of more mesh routers to achieve the best throughput performance. Optimal throughput is always achieved by using the minimum transmission range in inter-cell communications. In our model, it is equal to $l/\sqrt{m}$.

When the number of mesh routers is small, bottleneck is from intra-cell communications. Large ratio of clients’ number to routers’ number results in small throughput. Thus, in this case, adding mesh routers in the backbone network can effectively increase the throughput. When the number of mesh...
In MTPHC problem, the optimal throughput is evaluated by imposing constraint on the average number of hops in inter-cell communications. Fig.6 shows the per client throughput by varying the number of mesh routers under the condition of $\bar{h}(m,r) \leq 10$. With a loose constraint, e.g. $\bar{h}(m,r) \leq 6$ in this case, the optimal throughput is 42.2Kbits/s by deploying 196 mesh routers when assuming there are 500 mesh clients in the network. This is the same result as that in MTP problem. However, with tighter constraints, the results are very different from the results of MTP problem. The optimal throughput is 38.0Kbits/s and 29.7Kbits/s by deploying 81 and 64 mesh routers, respectively when $\bar{h}(m,r) \leq 6$ and $\bar{h}(m,r) \leq 4$. So, the trade-off between throughput and the number of hops has been observed. Fig.6 also demonstrates that adding excessive backbone nodes may dramatically degrade throughput performance in MTPHC problem. Therefore, when considering hops’ number constraint, it is much more critical to find a proper size of the backbone network for a WMN.

The results of MTPHC problem provide us another finding. In the case that we have less than optimal number of mesh routers, the idea always using the minimum transmission range may not be ideal. For example, in Fig.6, if the backbone network has equal or less than 64 mesh routers, it is better to adopt a larger transmission range, which is equal to $\sqrt{2l/\sqrt{m}}$. Comparing with using the minimum transmission range, i.e., $r = 1/\sqrt{m}$, the ideal one can achieve the same optimal throughput but less hops in the backbone network. Hence, MTPHC solution can also calculate the ideal transmission range when the backbone has less-than-optimal number of mesh routers.

VII. CONCLUSION

In this paper, a framework has been proposed to achieve non-asymptotic per client throughput of WMNs. We show that throughput of WMNs is constrained by both inter-cell and intra-cell communications. Deploying a proper number of backbone nodes can effectively balance the traffic load between these communications. Therefore, optimal throughput is obtained. When less-than-optimal number of backbone nodes is deployed, the ideal transmission range of backbone nodes can be also achieved by our solution. More sophisticated transmission, scheduling and routing schemes will be adopted in the future work.

REFERENCES


Fig. 5. Numeric results of MTP Problem

Fig. 6. Numeric results of MTPHC Problem, with $n=500$. 