In general, \( M \) is a random variable. From (8) and (9), we can write
\[
C_n^* = \Pr\{\text{SINR}(M) > b\}.
\]
(10)

Because \( P_{\text{rr},1}, P_{\text{rr},2}, \ldots, P_{\text{rr},n} \) are i.i.d. by assumption, it can be shown that, for \( i = 1, 2, \ldots, n \)
\[
\Pr\{\text{SINR}(1) > b, M = 1\} = \Pr\{\text{SINR}(i) > b, M = i\}.
\]
(11)

**Theorem 1:** Assume that the maximum received power level is unique, i.e., \( \{M = i\} \cap \{M = j\} = \emptyset \) for \( i \neq j \). This assumption holds when the received power levels are continuous random variables. Then
\[
C_n^* = n \Pr\{\text{SINR}(1) > b, M = 1\} \quad \text{for } b \geq 0.
\]

**Proof:**
\[
C_n^* = \Pr\{\text{SINR}(M) > b\}
= \sum_{i=1}^{n} \Pr\{\text{SINR}(i) > b, M = i\}
= \sum_{i=1}^{n} \Pr\{\text{SINR}(1) > b, M = 1\} \quad \text{[by (11)]}
= n \Pr\{\text{SINR}(1) > b, M = 1\}.
\]

**Remark 1:**

a) Theorem 1 is valid for all \( b \geq 0 \). Unlike expression (6), the correct expression for capture probability contains the event \( \{M = 1\} \). Note that
\[
\Pr\{\text{SINR}(1) > b, M = 1\} \leq \Pr\{\text{SINR}(1) > b\}
\]
because \( \{\text{SINR}(1) > b, M = 1\} \) is a subset of \( \{\text{SINR}(1) > b\} \). Then, it follows from Theorem 1 that:
\[
C_n^* \leq n \Pr\{\text{SINR}(1) > b\} = C_n
\]
for all \( b \geq 0 \). Thus, \( C_n \), given in (6) is an upper bound for capture probability. However, this upper bound is not very useful (because it can be much greater than 1 when \( b \) is much smaller than 1).

b) For the special case \( b > 1 \), the calculation of capture probability is further simplified as follows. From (7), when \( b > 1 \), we observe that the event \( \{\text{SINR}(1) > b\} \) implies the event \( \{M = 1\} \), i.e., \( \{\text{SINR}(1) > b\} \subseteq \{M = 1\} \). Thus,
\[
\{\text{SINR}(1) > b, M = 1\} = \{\text{SINR}(1) > b\}
\]
which implies
\[
\Pr\{\text{SINR}(1) > b, M = 1\} = \Pr\{\text{SINR}(1) > b\}.
\]
From Theorem 1, we then have
\[
C_n^* = n \Pr\{\text{SINR}(1) > b\}
\]
when \( b > 1 \). Thus, expression (6) for capture probability is correct when \( b > 1 \), i.e., \( C_n^* = C_n \) for this special case.

**REFERENCES**


**Reply to “Comments on ‘Capture and Retransmission Control in Mobile Radio’”**

Michele Zorzi and Ramesh R. Rao

In [1], we considered a single-cell radio access system based on slotted ALOHA. As usually done in the literature on capture ALOHA in wireless systems published until then, most of which we reference in our paper, we considered a narrowband system with a single antenna. Similar systems using code-division multiple access (CDMA) or spread spectrum techniques were usually referred to as “spread ALOHA,” whereas use of more than one antenna would be explicitly identified.] When such case is considered, the capture threshold \( b \) is necessarily greater than one, as it is not possible that two distinct packets are simultaneously captured.

While we implicitly referred to this scenario, we did not explicitly state this assumption, and we thank the authors of the Comment for identifying this issue and for giving us a chance to clarify.

In the case which was not addressed in [1], i.e., \( 0 \leq b \leq 1 \), there are two possible definitions for \( C_n^* \): 1) capture probability, i.e., the probability that at least one packet is successful in a collision of size \( n \) or 2) average number of successfully received packets in a collision of size \( n \) in the presence of a receiver capable of multiple receptions, i.e., all packets for which \( \text{SINR} > b \) can be simultaneously received (this latter definition of \( C_n^* \) is used in [2]). Definition 1) is also equivalent to the average number of successfully received packets in a collision of size \( n \) in the presence of a receiver capable of a single reception, i.e., even though multiple packets experience \( \text{SINR} > b \) only one of them can be received. In the case considered in [1], where \( b > 1 \), all these quantities are the same.

If, following [2], we define \( C_n \), as the average number of successes in a collision of size \( n \) with multiple reception capability, for any \( b \geq 0 \) the result is identical to the one given in [1]. In fact, let \( A_i \) be the event that packet \( i \)’s signal-to-interference ratio (SIR) is greater than or equal to \( b \), and let \( \mathcal{I}(A_i) = 1 \) if event \( A_i \) occurs, and 0 otherwise. Then, we have
\[
C_n = E\left[\sum_{i=1}^{n} \mathcal{I}(A_i) \right] \quad \text{\text{for } \text{packets}}
= n P[A_1]\text{\text{packets}}
\]
where \( P[A_1]\text{\text{packets}} \) is the same quantity \( P_r \) (conditioned on the collision size \( n \)) computed in [1, Sec. II]. In particular, as \( n \to \infty \), the

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limit $C_\infty = 2/(\pi \sqrt{S})$ of [1, Th. 1], as well as its more general expression given in [1, eq. (29)], still apply (as also reported by Hajek et al. in [3]).

If on the other hand we define $C_n$ as the probability that at least one packet can be captured in a slot (or equivalently, the throughput in the single-reception case), we have

$$C_n = P \left[ \bigcup_{i=1}^{n} A_i \right] \text{n packets}$$

$$= 1 - P \left[ \bigcap_{i=1}^{n} \overline{A}_i \right] \text{n packets}$$

(2)

where $\overline{A}$ is the complement of $A$. (This probability, though expressed differently, is the same as reported in the Comment.) In computing $C_n$ in this case one needs to account for the fact that the events $A_i$, $i = 1, 2, \ldots, n$ are dependent. For this reason, this computation cannot be derived directly from our analysis in [1], as correctly pointed out by the authors of the Comment, but would require a different approach, and is therefore to be seen as a separate contribution.

REFERENCES

