AN INTEGRATED AND DISTRIBUTED SCHEDULING AND POWER CONTROL ALGORITHM FOR MAXIMIZING NETWORK UTILITY FOR WIRELESS MULTIHOP NETWORKS

Jennifer C. Fang and Ramesh R. Rao
University of California, San Diego, La Jolla, CA.

ABSTRACT

In this paper, we solve the resource allocation problem of maximizing the sum of transmitter utilities subject to QoS and peak power constraints per link in a wireless multihop network. Each node in the network has an associated utility function that models its valuation of its data rate (or signal power) in terms of its transmission power and multi-access interference. By explicitly accounting for multi-access interference in the utility function, our framework can model and solve a wide variety of resource allocation problems. Each link in the network is subject to a minimum and a maximum data rate constraint and each transmitter is subject to a peak power constraint. We present an iterative power control algorithm that solves the above problem using a penalty function approach and prove its convergence to the optimal solution. Our power control policy is applicable over any subset of links scheduled. To achieve high data rates over the links in addition to maximizing system utility, we schedule links using a degree-based greedy algorithm that limits multi-access interference by scheduling a small number of transmissions around any scheduled receiver. The link scheduling algorithm and the power control algorithm are both amenable to distributed implementation in the framework of 802.11 LANs. Finally, we compare the performance of our joint scheduling and power control algorithms against CDMA using example utility functions and illustrate the superior performance of our algorithms.

I. INTRODUCTION

Wireless LANs today are capable of supporting data services through single hop communications using the Distributed Coordination Function in IEEE802.11. It has been shown by [3] that the use of multiple hops can greatly increase network capacity. Nodes in a multihop wireless network are, however, energy-limited and have a diverse set of requirements depending on the applications they run. Clearly, their valuation of bandwidth per unit Joule expended differ greatly. Utility functions have been used to model the valuation of bandwidth by user applications in wired and wireless networks to study application level adaptivity [4][8].

For example, MPEG video was modeled as a piecewise linear function [8]; the relative improvements in video quality with increasing bandwidth decreased monotonically. Utility functions have been shown to provide a framework to study congestion control and resource allocation through pricing in wired networks [4]. In [4], Steven Low et. al. consider a set of source nodes transmitting over a network composed of a set of finite capacity wired links to reach their corresponding destinations. They present an iterative rate control algorithm that maximizes the sum of utilities of the source nodes when subject to a flow conservation constraint on each link. Let the capacity of link $I$ be $C_I$. Defining $U_s(x_s)$ as the utility function of source $s$ as a function of its data rate $x_s$, where $U_s: \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}$, they compute the optimal rates $x^*_s$ that maximizes $\sum_{s \in S} U_s(x_s)$ subject to $\sum_{s \in E(l)} x_s \leq C_l$ using a duality approach.

In a wireless network, the rate of a link is a function of its signal-to-interference ratio and therefore depends on the powers of signal transmissions of all the users. The approach in [4] cannot be applied to multihop wireless networks, because there may not exist any feasible power vector that achieves the optimal rates $x^*_s$. In our approach, we model a node's utility as a function of all user transmission powers, thereby explicitly accounting for multi-access interference. Therefore we are able to compute the optimal power vector $P^*$ that maximizes the system utility subject to QoS constraints, whenever the problem is feasible. Examples of utility functions in a wireless network include the capacity and capacity per unit power of a link (or network), among others. We present an iterative and distributed power control algorithm that maximizes the sum of utilities of all transmitters in a multihop wireless network subject to minimum and maximum rate guarantees per link and a peak power constraint per transmitter. Our problem formulation is akin to QoS in ATM over wired networks, wherein each user is assured minimum and maximum rate guarantees. Minimum rate guarantees are important for high bandwidth applications like streaming audio, video etc. They are also useful in providing end-to-end delay guarantees for real time sessions. Maximum link rate constraints are necessary because of the limited buffering capabilities of portable wireless devices and the high sensitivity of transport protocols like TCP.
to loss due to buffer overflow in hybrid wired and wireless networks. For utility functions that are increasing functions of the SIR of a link, our power control algorithm also yields as a byproduct, the optimal schedule of links. For utility functions that account for multi-access interference in other forms, we find that joint scheduling and power control can increase the network throughput and utility beyond what is achievable without scheduling.

In section V, we present a greedy scheduling algorithm based on the nodal degrees that limits the total interference at the receivers by scheduling a small number of transmitters around them. Our power control and link scheduling algorithms can be easily implemented in the framework of CSMA/CA based MAC protocols like 802.11 DCF and MAC-RSV [7]. Our joint scheduling and power control algorithm achieve significantly higher data rates and system utility than current approaches like CDMA with and without power control as shown in section V.

II. SYSTEM MODEL

Consider a multihop wireless network with M stationary nodes sharing a common channel. Each transmitting node in the network could either transmit its own data or forward data for other nodes. We assume that the routes between nodes are already determined based on a routing strategy of choice. The power of signal transmission by node k is $P_k$. The signal attenuation between nodes $i$ and $j$ is denoted as $h_{ij}$. We assume $h_{ij}$ is constant for all node pairs. The signal to interference plus noise ratio, denoted by $\gamma_{ij}$, is

$$\gamma_{ij} = \frac{h_{ij}P_i}{\sum_{k=1,k\neq i}^{M} h_{kj}P_k + \sigma_j},$$

(1)

where $\sigma_j$ is the AWGN noise at node $j$. The link between any two nodes $i$ and $j$ is denoted as $e_{ij}$. We assume that transmitters use variable-rate M-QAM, with a bounded probability of symbol error and trellis coding with a nominal coding gain. Under these assumptions, the data rate of link $e_{ij}$ is denoted by $x_{ij}$ and is well approximated [3] by

$$x_{ij} = B_0 \log_2 \left(1 + \frac{\gamma_{ij}}{\Omega}\right)$$

(2)

where $B_0$ is the bandwidth of the channel, and $\Omega$ is the gap between uncoded M-QAM and the capacity, minus the coding gain. Henceforth, we use the boldface font for vectorial representation of powers, rates, etc, i.e., $P = \{P_1, P_2, \ldots, P_N\}$.

We assume that time is divided into slots of fixed length. Any valid scheduling policy must meet the following constraints:

- Each receiver can decode exactly one transmitter's data at any given time.
- Each node can only transmit to exactly one receiver at any given time.

The above constraints ensure half-duplex transmissions along links, thereby eliminating self-interference at receivers. Each node has an infinite amount of data to transmit, i.e., constantly backlogged. The utility function of node $i$ is a function of the transmit powers of all nodes in the system and is denoted by $U_i(P)$. Of course, each node can only control its own transmission power. Each node is subject to a peak transmit power constraint of at most $P_{\text{max}}(i)$. Finally, we assume that $\sum_{i \in M} U_i(P)$ is continuous, twice differentiable and strictly concave in $P$ and the hessian of $\sum_{i \in M} U_i(P)$ is bounded.

III. THE RESOURCE ALLOCATION PROBLEM

For a set of links scheduled by any valid scheduling policy, we maximize the sum of utilities of these links through power control. Let the set of active transmitters and receivers in a given slot be denoted by $T$ and $R$, respectively. Let $N$ be the number of transmitting-receiving node pairs. We denote the intended receiving node of transmitting node $i$ by $r(i)$. Our objective is to find an optimal power allocation policy that maximizes the sum of utilities of all transmitting nodes subject to a minimum and a maximum rate constraint per link and a peak power constraint for each transmitter. Mathematically speaking, we

$$\max_{i \in T} \sum_{i \in T} U_i(P) = \min \left\{ \sum_{i \in T} U_i(P) \right\},$$

(3)

subject to the quality of service (QoS) constraints

$$C_r^{\text{min}}(i) \leq x_{ir(i)} \leq C_r^{\text{max}}(i), \quad \forall r(i) \in R,$$

(4)

$$0 \leq P_i \leq P_{\text{max}}(i), \quad \forall i \in T,$$

(5)

where $C_r^{\text{min}}(i)$ and $C_r^{\text{max}}(i)$ are the minimum and maximum supported capacity by node $r(i)$ for link $e_{ir(i)}$, respectively. The problem (3–5) is a convex optimization problem and has a unique optimal solution, because $\sum_{i \in T} U_i(P)$ is strictly concave and the feasible region is convex and compact. A centralized solution to the above problem is computationally intensive and requires excessive coordination among users. Instead, we solve the above problem in a distributed manner using a penalty function approach.

Constraint (4) can be rewritten as a linear constraint in $P$, since $C_r^{\text{min}}(i) \leq x_{ir(i)} \leq C_r^{\text{max}}(i)$ can be expressed as $\gamma_{ir(i)}^{\text{min}} \leq \gamma_{ir(i)} \leq \gamma_{ir(i)}^{\text{max}}$ or,

$$\gamma_{ir(i)}^{\text{min}} \leq \frac{h_{ir(i)}P_i}{\sum_{k \in \{T-i\}} h_{kr(i)}P_k + \sigma_{r(i)}} \leq \gamma_{ir(i)}^{\text{max}},$$

where $\gamma_{ir(i)}^{\text{min}}$ and $\gamma_{ir(i)}^{\text{max}}$ are the minimum and maximum SIR of link $e_{ir(i)}$, respectively.
which can be rewritten as

$$\sum_{k \in T} \alpha^A_{kr(i)} P_k \leq \sigma_r(i) \quad \text{and} \quad \sum_{k \in T} \alpha^B_{kr(i)} P_k \geq \sigma_r(i)$$

\(\forall r(i) \in R\), respectively, where

$$\alpha^A_{kr(i)} = \begin{cases} h^A_{rr(i)} & k = i \\ -h^A_{kr(i)} & k \neq i \end{cases}$$

and

$$\alpha^B_{kr(i)} = \begin{cases} h^B_{rr(i)} & k = i \\ -h^B_{kr(i)} & k \neq i \end{cases}.$$

We mention here that Bambos et al. [6] present a distributed iterative algorithm that computes the minimal power vector required to support a minimum rate guarantee per link in a multihop wireless network. Our resource allocation problem bears similarities to [6] but maximizes a more general class of objective functions by incorporating the concept of user utility in terms of rate and transmission power and accounting for interference. Our problem also includes an additional restriction on the maximum supportable data rate along links. Additionally, we find that scheduling links using our proposed greedy degree-based scheduling algorithm (section V) can achieve high link rates while maximizing system utility, thus eliminating the need for call admission control as described in [6]. We also found that the duality approach in [4] can be applied to solve problem (3-5) only when links are scheduled sufficiently far away from each other. The following section describes our approach.

A. A PENALTY FUNCTION APPROACH

We solve the constrained optimization problem in (3-5) through a series of unconstrained minimizations of a modified objective function \(L(P, q)\) obtained by adding a penalty term to the primal objective function \(\sum_{i \in T} -U_i(P)\):

$$L(P, q) = \left\{ \sum_{i \in T} -U_i(P) \right\} + \sum_{r(j) \in R} \frac{q}{2} \left[ \max \left\{ 0, \sum_{i \in T} \alpha^A_{r(j)i} P_i - \sigma_{r(j)} \right\} \right]^2 \quad \text{and} \quad \sum_{r(j) \in R} \frac{q}{2} \left[ \max \left\{ 0, \sigma_{r(j)} - \sum_{i \in T} \alpha^B_{r(j)i} P_i \right\} \right]^2.$$

For a given value of \(q > 0\), the modified objective function \(L(P, q)\) represents the negative of the sum of transmitter utilities, plus the cost of violating any of the constraints. It is an upper bound to any feasible solution for the primal problem (3-5). As shall be shown later, \(L(P, q)\) is lower bounded, continuously differentiable and strictly convex in \(P\) over the compact set \(0 \leq P \leq P_{\max}\) for a given value of \(q\). The solution to the primal problem (3-5) is obtained through a series of sequential minimizations of \(L(P, q_k)\) over the compact set \(0 \leq P \leq P_{\max}\) for an increasing sequence \(\{q_k \mid q_k > 0, k \geq 1\}\). For each \(q_k\), define \(P^*(q_k)\) as the optimal solution of the following problem

$$P^*(q_k) = \arg \min_{0 \leq P \leq P_{\max}} \{ L(P, q_k) \}. \quad (7)$$

The solution to the primal problem (3-5) is then \(P^{opt}\), where

$$P^{opt} = \lim_{q_k \to \infty} P^*(q_k).$$

Refer to Theorem 1 for proof. We remark here that we could potentially solve (7) for \(P^*(q_k)\) by setting the first derivative of \(L(P, q_k)\) to zero and solving the system of \(N\) equations. However, since it is difficult to obtain a closed-form expression this way, we employ an iterative descent algorithm. For each element in the sequence \(\{q_k\}\), the descent algorithm iteratively finds an optimal solution \(P^*(q_k)\). A desirable feature of using the iterative descent algorithm is its suitability for distributed implementation. In the following section, we describe the iterative descent algorithm that solves (7), i.e., minimizes \(L(P, q_k)\) for a given \(q_k > 0\).

IV. A DISTRIBUTED POWER CONTROL ALGORITHM

We now present a synchronous distributed iterative algorithm for each user to compute the optimal transmission power in order to maximize the system utility. Assuming that \(\sum_{i \in T} U_i(P)\) is strictly concave in \(P\), we can show that \(L(P, q_k)\) and its gradient have the following properties (see lemmas 2–3 in Appendix for proofs):

1) \(L(P, q_k)\) is lower bounded, continuously differentiable and strictly convex in \(P \in [0, P_{\max}]\) for all \(q_k > 0\).
2) \(\nabla L(P, q_k)\) is Lipschitz continuous.

Each transmitter \(i\) uses the following gradient projection method to compute its transmission power iteratively for each value of \(q_k > 0\).

**User i's Computation for \(q_k > 0, t = 1, 2, \ldots\)**

1) Compute optimal power level for \(t + 1:\)

$$P_t(t + 1) = \left[ P_t(t) - \Delta \frac{\partial L(P_t(t), q_k)}{\partial P_t} \right]_{P_{\max}(t)}.$$
where
\[
\frac{\partial L(P(t), q_k)}{\partial P_i} = \sum_{m \in T} \{-U_m(P(t))\} + q_k \sum_{r(j) \in R} \alpha^A_{r(j)} \max \left\{ 0, \sum_{m \in T} \alpha^A_{mr(j)} P_m(t) - \sigma_{r(j)} \right\} - q_k \sum_{r(j) \in R} \alpha^B_{r(j)} \max \left\{ 0, \sigma_{r(j)} - \sum_{m \in T} \alpha^B_{mr(j)} P_m(t) \right\}.
\]

For each \(q_k > 0\), we prove that \(\lim_{t \to \infty} P_i(t)\) exists and is equal to \(P^*_i(q_k)\) for all \(i \in T\). Since \(P^*_i(q_k)\) exists for \(0 < \Delta < 2/K_q\) (\(K_q\) is a constant defined in lemma 3), then by [Prop 3.4, [1]], it is the solution of \(\min L(P, q_k)\).

Note that our algorithm does not involve any receiver computations. However, each receiver \(r(i)\) needs to broadcast a scaled value of \(x_{r(i)}(t), U'_i(P(t))\) and \(P_i(t)\) to all scheduled transmitters. The sequence \(q_k\) can be different for each receiver and can be updated at different frequencies. However, it is reasonable to assume that all nodes use the same sequence \(q_k\) and update their values at the same time, since they could learn its value from infrequent broadcasts. Scheduling a limited number of transmissions in any neighborhood can significantly reduce the amount of control information broadcasted by receivers. The following theorem proves the convergence of our power control algorithm as \(q_k \to \infty\).

**Theorem 1:** Let \(\{q_k\}, k = 1, 2, \ldots\) be a non-negative increasing sequence. Let \(\{P^*_k(q_k)\}\) be a sequence generated by the penalty method. Then any limit point of the sequence is a solution to the primal problem (3-5), i.e.,

\[
P^{opt} = \lim_{q_k \to \infty} P^*_k(q_k).
\]

**Proof:** The convergence of our power control algorithm follows from lemmas 2–3 and the theorem in [p.367, [2]].

The iterative power control algorithm converges rapidly to the optimal solution and gets within an \(E\) precision of the optimal solution \(L(P^*(q_k), q_k)\) in a few iterations. This is particularly relevant in real systems where the signal transmission power is restricted to integer multiples gauged in milliwatts. By computing \(E\)-optimal solutions for each subproblem (7), our power control algorithm converges to an \(E\)-optimal solution to the primal problem (3–5) in a finite number of iterations. This is restated in the following corollary, but the proof is omitted due to space limitation.

**Corollary 1:** Suppose the solution to the problem \(\min \{L(P, q_k)\}\) is within \(\mathcal{E}\) of the optimal solution for each \(q_k > 0\), then the solution to problem (3) is \(\mathcal{E}\)-optimal.

Our power control policy maximizes the sum of transmitter utilities for any scheduled subset of links. However, when user utility functions do not account for multi-access interference, the data rate achieved using the optimal power control policy could be low due to a large number of interfering transmissions. Scheduling link transmissions and then computing their utility maximizing powers using the algorithm described in section IV can greatly increase link rates. Joint scheduling and power control is also rewarding for the class of utility functions which account for multi-access interference explicitly, since the data rate and utility are decreasing in multi-access interference. An important consequence of scheduling transmissions is that it limits the amount of feedback information necessary for our iterative power control algorithm. In the following section, we describe a degree-based greedy scheduling algorithm that limits multi-access interference at any scheduled receiver.

V. SCHEDULING MECHANISM

The degree-based greedy scheduling algorithm computes an approximate maximum independent set of links in a multihop network. Given a network of \(M\) nodes, we construct a graph \(G\) such that any node within a 1-hop sensing range of another node has an edge with that node. Let \(D(i, j)\) denote the distance between node \(i\) and node \(j\). The 1-hop sensing range of a receiving node \(r(i)\) is assumed to be \(D(i, r(i)) + R\), where \(R\) is a system specified constant. Let the set of 1-hop neighbors of node \(i^*\) be \(N(i^*)\). The degree of node \(i^*\) is \(|N(i^*)|\).

The scheduling algorithm pre-selects a transmitter node \(i^*\) and then finds a transmitter node \(j^*\) with lowest degree in the subgraph \(\{G - i^* - N(r(i^*))\}\). It then finds the next least degree transmitter node \(k^*\) in the subgraph \(\{G - i^* - N(r(i^*)) - j^* - N(r(j^*))\}\) and so on, until all transmitters are covered. The resulting subset of transmitter nodes is an approximate MIS of transmitters in the given network with an approximation ratio \(\Delta^2/2\), where \(\Delta\) is the maximum nodal degree in \(G\) [10]. A natural consequence of this heuristic is that it limits the number of interfering transmitters near any receiving node of an active link. It also lends itself to a straightforward implementation in the framework of IEEE 802.11 and MAC-RSV [7] by suitable modification of the backoff mechanism. To see this, let node \(i^*\) transmit an RTS in the \(K_{i^*}\) minislot in the channel contention window, where

\[
K_{i^*} = |N(i^*)| + V,
\]

where \(V\) is a random variable in \((0,1)\). Transmitters with low degrees will win the channel for the corresponding data slot. The random variable \(V\) is introduced to minimize collisions between neighboring transmitters with the same degree. Upon hearing a CTS from \(r(i^*)\), all transmitters within
the sensing range of node $r(i^*)$ will remain silent (cease contention) for the subsequent data slot. The heuristic would thus schedule transmitters with low nodal degrees, such that they are sufficiently far away from other receivers. It is easy to see that by increasing $R$, the above scheduling algorithm eliminates the hidden and exposed node problem that plagues 802.11 LANs.

To ensure all transmitters are scheduled at least once every few slots, each transmitter is preseleced by the heuristic at least once. A fair allocation of resources can be achieved by controlling the frequency of preselection. An alternative mechanism to ensure minimum service to each transmitter is to force them to back-off from channel contention by increasing their respective $K_i^*$ by a fixed value immediately after a successful transmission. We now derive the maximum number of scheduled transmitters close to any receiver. Using the triangular inequality, we can show that

$$D(r(i), r(k)) \geq D(r(i), i) - D(i, r(k)) \geq R,$$

since $D(i, r(k)) \geq D(r(i), i) + R$. The distance between any two scheduled receivers is at least $R$. Consider a scenario where all the scheduled receivers are as close to each other as possible, i.e. $D(r(i), i) = R$. This is in some sense a worst-case scenario, since the interfering transmitters are very likely to be in close proximity to a given receiver. Using simple geometric arguments the following lemma can be proven to bound the number of scheduled receivers around any given receiver.

**Lemma 1:** Let $C_0$ be a circle of radius $r_0 - \epsilon_0$ and let $S$ be a set of circles of radius $\epsilon_0$ such that every circle in $S$ intersects $C_0$ and no two circles in $S$ intersect each other at more than one point. Then $|S| \leq 6$.

**VI. RESULTS AND OBSERVATIONS**

In this section, we illustrate the fast convergence of our power control algorithm for any valid set of scheduled links. Then, we compare the performance of our scheduling and power control algorithm to CDMA implementing our power control algorithm. For the convergence experiment, we consider a topology consisting of 10 links and 20 randomly placed nodes in a 100$m \times 100$m field as shown in Fig. 1. The signal attenuation is modeled as the inverse square of the distance between nodes. The utility function for this experiment denotes the Shannon capacity of a link, i.e., $U_i(P) = \log_2(1 + h_{ir(i)}P_i)$. The sequence $\{q_k\}$ is assumed to be the same for all the nodes and is given by $\{q_k = k: k \geq 1\}$. The peak transmission power for each node is identically set to 500 milliwatts, $C_r^{\text{min}} = 0$ and $\sigma_{r(i)} = 0.5$ mW. Fig. 2 plots the convergence of the iterative power control algorithm with increasing values of $q_k$. The powers of nodes 3, 5, 7, 9, and 19 converges rapidly (with only 5 values of the sequence $q_k$) to their optimal solution as shown in Fig. 2; the powers of other nodes not shown in this figure, converge similarly.

Next, we compare the performance of our joint scheduling and power control algorithm to a base policy (CDMA) that implements our optimal power control algorithm alone over large networks. The utility function for this experiment was

$$U_i(P) = A_0P_i \exp(-A_1h_{ir(i)}P_i) + A_2(h_{ir(i)} - \sum_{m \neq i} h_{ir(m)})P_i,$$

where $A_0$, $A_1$ and $A_2$ are constants. The first term in $U_i(P)$ increases with $P_i$ for $P_i \leq P_i^*$, and decreases monotonically for $P_i > P_i^*$, where $P_i^* = 1/A_1h_{ir(i)}$. We remark here that the total energy (circuit power plus signal power) required to transmit a single bit is convex and increasing in the data rate [9]. The energy efficiency of transmitting at $P_i$ therefore increases at low powers but decreases at high powers due to the concave dependence of data rate with transmission power and the convex and increasing power necessary to achieve high rates. The first term in this utility function models energy efficiency of a transmission and is similar in profile to the energy efficient utility function in [5]. We account for interference in the second term in $U_i(P)$. This term models the net increase in utility when a transmitter has a strong channel (thereby capable of high rates) and does not interfere with other ongoing transmissions. Since the cost of interfering is high, transmitters in dense neighborhoods will transmit at low powers.

We ran our joint scheduling and power control algorithm for a 40 node, 20 link network and compare its performance against a base policy which implements our power control alone, as the average number of scheduled links in the network decreases. The degree of a receiver is increased by increasing the parameter $R$, discussed in section V. Note that the number of simultaneous transmissions decreases with increasing receiver degree. For this experiment: $P_{\text{max}} = 200$ mW, $\epsilon_0 = 1$mW, and $A_0 = A_1 = 1$ and $A_2 = 2$. The sum $\sum_{i \in S} U_i(P)$ is strictly concave in $0 < P \leq P_{\text{max}}$. Nodes are randomly distributed over a $50m \times 50m$ region. There are a total of 20 disjoint transmitting-receiving pairs (links). Each node has a transmission has a range of 10m. In the base policy, all links are active at the same time with powers $P_i^*$ determined by our optimal power control policy.

Fig. 3 plots the ratio of utilities achieved by the joint scheduling and power control policy to the base policy. At
low receiver degrees, our joint scheduling and power control algorithms achieve significantly higher system utility than the base policy. This is due to the limited multi-access interference at receivers, achieved by scheduling links using our scheduling policy. However, the relative gains in utility decrease monotonically as the average degree of the network increases. This is attributed in part to the conservative use of the wireless spectrum, with only a few simultaneous transmissions. Another factor contributing to the decrease in utility gains is the relative dominance of the first term in $\sum_{t \in T} U_t(P)$ which is independent of multi-access interference. The scheduling policy activates few transmitters and although they achieve high utilities, their relative sparsity at high receiver degrees results in little utility gain over the base policy. As we decrease the weight $A_0$ in $U_t(P)$, the gains of joint scheduling and power control increase significantly over the base policy.

Fig. 1. Network topology.

Fig. 2. Convergence of the link powers to the optimal values.

We plot the ratio of the sum of link rates achieved by joint scheduling and power control to the base policy in Fig. 4, with increasing receiver degree. We find that for low receiver degrees the sum of link rates increases but starts decreasing at sufficiently high degrees. This is so because at low receiver degrees, there are too many simultaneous transmissions resulting in high multi-access interference. At high receiver degrees, we find that the number of simultaneous transmitters are very few, resulting in a suboptimal use of the wireless spectrum. We find that through scheduling and power control we are able to achieve higher rates than the base policy, while consuming lower power, making the network operation highly energy efficient. The improvements in throughput and utility of our policy over pure CDMA without power control are orders of magnitude higher. We observed similar results as in Figures 3 and 4 for a wide range of system parameters and network topologies. We believe that our scheduling algorithm eliminates the hidden and exposed node problems that plague 802.11 DCF, and can therefore achieve higher rates than 802.11’s scheduling policy. For the class of utility functions which are increasing functions of the SIR of a link, we find the our power control algorithm also computes the optimal transmission schedule of links.

VII. CONCLUSION

We solve the resource allocation problem of maximizing the sum of transmitter utilities subject to a minimum and maximum data rate constraint per link and peak power constraints per node in a wireless multihop network. We consider a general class of transmitter utility functions that could
account for multi-access interference explicitly. Our iterative power control algorithm converges very rapidly to the optimal solution, regardless of the link scheduling policy. Our power control algorithm with simple modifications can be used to solve an alternative resource allocation problem where arbitrary subsets of nodes are subject to a total transmit power constraint. The rapid convergence of our power control algorithm makes it suitable for implementation over time-varying block fading wireless channels while maintaining optimality. Although not shown in this paper, our power control algorithm converges in an asynchronous environment as well. To achieve high link rates and utilities, we have developed a link scheduling algorithm that schedules transmissions using receiver degree information. The link scheduling algorithm is simple and effective in limiting interference and is easily implementable in the framework of CSMA/CA MAC protocols like 802.11 DCF. The improvements in network throughput and system utility through scheduling and power control over conventional policies like CDMA and TDMA are significant.

VIII. APPENDIX: CONVERGENCE OF THE ALGORITHM

Lemma 2: \( L(P, q) \) is strictly convex in \( P \) for any \( q > 0 \).

Proof: Since \( \{\sum_{t \in T} -U_t(P)\} \) is strictly convex in \( P \), its hessian \( H \) is positive definite. Let the Hessian of the penalty function be \( H \) for a fixed \( q > 0 \). We need to show that \( H \) is positive definite, i.e., \( P^T H P > 0 \) for all \( P \). The elements of the Hessian matrix are

\[
\frac{\partial^2 L(P, q)}{\partial P_i^2} = H_u(i, i) + q(\sum_{r_{ij} \in R} (a_{ir_{ij}}^A)^2 I_{ir_{ij}}^A) + \sum_{r_{ij} \in R} (a_{ir_{ij}}^B)^2 I_{ir_{ij}}^B,
\]

\[
\frac{\partial^2 L(P, q)}{\partial P_j \partial P_k} = H_u(i, k) + q(\sum_{r_{ij} \in R} \alpha_{ir_{ij}}^A \alpha_{kr_{ij}}^A I_{ir_{ij}}^A) + \sum_{r_{ij} \in R} \alpha_{ir_{ij}}^B \alpha_{kr_{ij}}^B I_{ir_{ij}}^B,
\]

where \( I_{ir_{ij}}^A \) and \( I_{ir_{ij}}^B \) are indicator functions equal to 1 whenever constraints (6) are violated, respectively.

Let \( [A]^A \) and \( [B]^B \) be \( N \times N \) matrices with zero elements except the entries \( A_{ir_{ij}}^A = 1 \) and \( B_{ir_{ij}}^B = 1 \) whenever the \( I_{ir_{ij}}^A = 1 \) and \( I_{ir_{ij}}^B = 1 \), respectively. Defining \( A_u = [A]^A \alpha_A^A \) and \( A_b = -[B]^B \alpha_A^B \), we can rewrite \( H \) as

\[
H = H_u + q[A]^T A_u + q[A]^T A_b
\]

Thus, the Hessian is positive definite, and therefore \( L(P, q) \) is strictly convex in \( P \).

It is worth noting that the Hessian of \( L(P, q) \) is positive definite, although it may contain negative off-diagonal elements.

Lemma 3: \( \nabla L(P, q) \) is Lipschitz continuous.

Proof: From Mean Value Theorem,

\[
\|\nabla L(P, q) - \nabla L(P_0, q)\|_2 \leq \|L(P_0, q)\|_2 \cdot \|P - P_0\|_2 \leq \|L(P_0, q)\|_\infty \cdot \|P - P_0\|_2
\]

\[
\leq |L + q\alpha_{ir_{ij}}^1| \cdot \|P - P_0\|_2 = K_q \|P - P_0\|_2,
\]

where \( K_q \) is a constant, \( P_0 = tP_a + (1 - t)P_b, t \in [0, 1] \). \( \alpha_{ir_{ij}} \) is the maximum row sum of \( [a^1]^T [a^A] + [a^B]^T [a^B] \) and \( \|H_u\|_2 \leq L \) and is assumed to be bounded. Note that since \( \|L(P_0, q)\|_2 \leq \|L^2(P_0, q)\|_\infty \cdot \|L^2(P_0, q)\|_1 \), and \( \|L(P_0, q)\|_2 \) is symmetric, we have \( \|L(P_0, q)\|_\infty = \|L^2(P_0, q)\|_1 \), hence the second inequality.

Theorem 2: Let \( 0 < \Delta < 2/K_q \). Denote \( P^*(q) \) as the limit point for the sequence \( \{P(t)\} \) generated by the algorithm in IV for a given \( c > 0 \). If \( L(P, q) \) is lower bounded, continuously differentiable, Lipschitz continuous, and in particular, convex in \( 0 \leq P \leq P_{\text{max}} \), \( P^*(q) \) minimizes \( L(P, q) \) in \( 0 \leq P \leq P_{\text{max}} \).

Proof: See [p.214, [1]].

REFERENCES