Delay Analysis of Block Coded Transmission over the Gilbert-Elliott Channel with Interleaving and Retransmission Strategy*

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Abstract- To determine the end-to-end delay in block coded transmission over the Gilbert-Elliott channel with interleaving and a retransmission strategy, the probability distribution of the number of transmissions that a packet has to go through before it succeeds has to be found. We propose an approximate method to evaluate the distribution when the error statistics are dependent. The end-to-end delay is then examined. Our study provides a way to choose an error-control scheme (i.e., an interleaver of an appropriate size with a retransmission protocol), which meets both the delay constraint and a reliability requirement.

1 Introduction

A major task in data communications is how to control transmitted errors caused by channel noise so that error-free data can be delivered to the user. This task becomes more complex when multimedia applications, with delay-sensitive information and varying reliability requirements, are carried through the network.

In a wireless environment, where the channel typically exhibits memory, assuming forward error correction is employed, an interleaver is used to render the symbols less dependent. However, the delay added by the interleaver can exceed the delay constraint on a particular application. On the other hand, if we reduce the interleaver size by decreasing the interleaving depth to meet the delay constraint, the reliability of the system may suffer. A possible solution is to employ additional retransmission schemes to compensate for the degradation of the reliability when the interleaver size is reduced. Since it is possible that the resulting decrease in the interleaver is larger than the delay introduced by the retransmissions, both the reliability and the delay constraint can be met.

In order to determine the delay introduced by retransmissions, the probability distribution of the number of transmissions that a packet has to go through before it succeeds has to be found. In situations where memory is accounted for, there are few analytical results, and such a probability distribution is therefore often obtained via simulations.

In this work, we focus on block coded transmission over the Gilbert-Elliott (GE) channel [1][2]. In this model, the channel is assumed to be either in a good state, where the probability of error is small, or in a bad state, where the probability of error is significantly larger. The dynamics of the channel are modeled as a first-order Markov chain, a model which Wang and Moayer [3] and Wang and Chang [4] showed to be very accurate for a Rayleigh fading channel. In [5], Wilhelmsson and Milstein gave an analytical expression for the probability of codeword error on an interleaved GE channel when a t-error correcting block code of length n is used for error correction. In this paper, we propose a retransmission protocol, and obtain an approximate expression for the probability of the number of transmissions.

The remainder of the paper is organized as follows. In Section 2, we describe the system model, with a brief review of the GE channel model in Subsection 2.1 and a detailed description of the retransmission protocol in Subsection 2.2. Section 3 gives an approximate expression for the probability of the number of transmissions. In Section 4, results obtained by simulation and the analysis in Section 3 are compared and analyzed. Finally, conclusions can be found in Section 4.

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2 System Model

We study block coded transmission over a fading channel in this paper. The original message is BCH encoded and interleaved before it is passed into the channel. The interleaver we are using is a block interleaver with \( m \) rows and \( n \) columns, where the bits that are to be transmitted are fed in row-wise and fed out column-wise. It is known that the effectiveness of an interleaver depends on the value of \( m \), the interleaving depth. Typically, the larger value of \( m \), the better the interleaver can be expected to work in a channel with memory. However, under a delay constraint, the size of the interleaver has to be limited. In order to maintain the reliability of the system, we employ a retransmission protocol, a description of which is given in Subsection 2.2.

2.1 GE Channel

The GE channel is a first-order, discrete stationary, Markov chain with two states, one good and one bad, denoted by "1" and "0", respectively. In order to describe the channel, the following notation is used: The probabilities that the channel state changes from "1" to "0" and from "0" to "1" are denoted by \( b \) and \( g \), respectively.

With an original GE channel and an interleaver with interleaving depth \( m \), the equivalent GE channel will have the corresponding transition probabilities \( b' \) and \( g' \), given by [5]

\[
b' = \frac{b}{b + g} (1 - (1 - b - g)^m) \quad (1)
g' = \frac{g}{b + g} (1 - (1 - b - g)^m). \quad (2)
\]

2.2 Retransmission Protocol Description

The transmitter continuously transmits codewords in order and then stores them in a queue, called a retransmission queue (R.Q.). The transmitter has two transmission modes: one is the ordinary mode, and the other is the retransmission mode. The transmitter stays in ordinary mode as long as the R.Q. is empty. Fig 2 depicts the operations of the transmitter when it is in the ordinary mode.

If the retransmission queue is not empty, the transmitter enters retransmission mode. In this mode, the transmitter puts the first codeword of the queue into the first row of the interleaver (if the interleaver is not empty, then the transmitter waits until the interleaver is empty). The remaining rows of the interleaver are filled with new codewords. We say that the codeword in the first row is in retransmission service, and it is denoted as codeword \( A \). During the interval between the transmission of codeword \( A \) and the receipt of an acknowledgement for it, the transmitter continues to put new information packets into the interleaver and transmits them. When an ACK for codeword \( A \) is received, codeword \( A \) is removed from the retransmission queue, and the transmitter is ready to serve the next packet in the retransmission queue until the queue is empty. If a NAK for codeword \( A \) is received, the transmitter continues to be in the retransmission service for codeword \( A \) until an ACK is received. All codewords other than codeword \( A \) which are NAKed are put into the retransmission queue. When the retransmission queue is cleared, the transmitter resumes normal transmissions. Fig 3 shows the flow graph of the retransmission mode. Note that we assume we do not allow any packet drop in our protocol description here. It is easy to modify the protocol by setting a threshold on the number of retransmissions such that if the number of retransmissions for any codeword exceeds the threshold, that codeword is dropped and therefore a codeword error occurs.

3 Analysis

Due to the bursty nature of the channel, the errors are not independent from packet to packet. The number of transmissions needed to transmit a packet error-free is no longer a geometric distribution. Since the second order statistics are unknown, a closed form expression for the distribution is difficult to determine. However, with the help of some assumptions (which are detailed below), we are able to arrive at the following approximate expression:

**Theorem 1.** If a \( t \)-error correcting block code of length \( n \) is used for error correction when the transmission is done over the interleaved Gilbert-Elliott channel, with the retransmission scheme described in Subsection 2.2, the probability of the number of transmissions needed to transmit a packet through the channel error-free is
approximately given by:

\[ P(I = t) = \sum_{t = 0}^{1} \prod_{i=1}^{t} \left[ \sum_{j=0}^{1} (1 - P_j) K_j \right] \]

\[ \left[ \sum_{d_i=1} P_{i-1} K_{i-1} \right] \cdots \left[ \sum_{d_j=2} P_{j} K_{j} \right] \left[ \sum_{d_l=1} P \right] K_{l} \]

where

\[ P_j = P(e_{S_j} + e_{B_j} > t | D_j = d_j) \]

\[ = \sum_{e_{S_j} = 0}^{d_j} \left( P(e_{S_j} | d_j) \sum_{n=d_j}^{n-1} P(s_{S_j}) \right) \]

for all \( i \),

\[ K_j = \left\{ \begin{array}{ll}
P(D_j = d_j | S_{j-n} = s_j, S_{j-n-1} = s_{j-1}) & j = 1 \\
\sum_{j=1}^{n} P(e_{B_j} | d_j) & j \neq 1
\end{array} \right. \]

and each term in \( K_j \) can be easily obtained from Theorem I in [5] (applying Bayes rule).

The variables are defined as follows:

1. \( I \) is the number of transmissions needed to transmit a packet error-free.

2. \( S_{j-n}, j = 1, 2, \ldots, i-1 \), is the state of the channel at the instant of time when the last bit of the \( j \)th transmitted copy of the packet is transmitted. It takes the value of 0 or 1, corresponding to the channel being in the Good state or the Bad state, respectively. \( S^* \) is the state of the channel at the instant of time when the first bit of the first duplicate copy of the retransmitted packet is transmitted.

3. \( D_j = \sum_{i=1}^{n} (e_{B_j} + e_{S_j}) \) is a random variable that counts the number of times the channel stays in the Bad state during the transmission of one packet.

4. \( e_{B_j} \) and \( e_{S_j} \) denote the number of bits in error when the channel is in the Good state and the Bad state, respectively during the \( j \)th transmission of the packet.

5. \( P_{p+1} \) is the transition probability of the underlying Markov chain, which can be obtained from (1) and (2).

6. \( P(e_{B_j} | d_j) \) and \( P(e_{S_j} | d_j) \) are the probabilities of \( e_{B_j} \) and \( e_{S_j} \) errors in the Bad state and the Good state, respectively, conditioned on \( d_j \). They are given by [5]

\[ P(e_{B_j} | d_j) = \left( \frac{d_j}{e_{B_j}} \right) P_{S_j} (1 - P_{S_j})^{e_{B_j}} \]

and

\[ P(e_{S_j} | d_j) = \left( \frac{n}{e_{S_j}} \right) P_{G_j} (1 - P_{G_j})^{n-d_j-e_{S_j}}. \]
The above approximation is proven by finding the distribution conditioned on the channel states when the first bit and the last bit of each copy of the packet are transmitted, and then averaging over all possible channel states. The approximation consists of assuming that the codeword error (packet error) probability only depends on the number of times the channel stays in the Bad state during the transmission of the codeword.

4 Results

In order to determine the goodness of the approximation, we compare simulation results with those obtained from the approximation. The following parameters are used in the simulation:

- carrier frequency=1.8 Ghz;
- information rate 9.6 kbit/s;
- BPSK modulation;
- FEC uses a (15,7,2) BCH code;
- interleaver depth $m = 27$;
- pico cell size ($r=1000m$);
- normalized Doppler frequency = 0.003 (corresponding to a vehicle speed of roughly 20 mph)

We match the parameters of the GE channel to the land mobil channel, as in [5]. Fig 4 shows a good match between the simulation results and those we obtain through Theorem 1.

Also, we use the approximation to determine the delay introduced by the retransmission, and then we find out the total end-to-end delay. Table 1 lists the total delay corresponding to different interleaver depths. Here Pico stands for a pico cell with a radius 1km, LEO stands for a low altitude satellite with an orbit roughly at 2000km, and GEO stands for a geostationary satellite with an orbit approximately at 36000km. In a system without a retransmission strategy, for $m=27$, the total end-to-end delay is $19.70$ ms, $26.4$ ms and $140$ ms respectively. Comparing with Table 1, it is clearly seen that we are better off in terms of minimizing delay if we use a retransmission scheme alone without any interleaving in a Pico cell and a LEO environment. However, in the GEO case, an interleaved system without retransmission yields smaller delay. Fig 5 shows the packet loss probability for different interleaving depths. The result is obtained by setting the threshold to 16, i.e., if a packet needs more than 16 retransmissions, it is dropped and a packet loss is declared. We notice that the reliability degrades in both the Pico cell and LEO case when the interleaving depth is reduced. This degradation can be compensated by increasing the value of the threshold, or by increasing the interleaving depth as long as the overall delay does not exceed the delay constraint.

Another interesting observation is that when we increase the data rate (for example, to 1M bits/s), with all other parameters fixed, we need a substantially larger interleaver to improve the system reliability, and this introduces a large delay.
Table 1: Total end-to-end delay in ms with the parameters defined above.

<table>
<thead>
<tr>
<th>Cell Type</th>
<th>Pico</th>
<th>LEO</th>
<th>GEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR(db)</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>m=27</td>
<td>19.7</td>
<td>19.7</td>
<td>19.7</td>
</tr>
<tr>
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<tr>
<td>m=5</td>
<td>3.65</td>
<td>3.65</td>
<td>3.65</td>
</tr>
<tr>
<td>m=1</td>
<td>0.74</td>
<td>0.74</td>
<td>0.74</td>
</tr>
</tbody>
</table>

5 Conclusion

The choice between using or not using an interleaver depends on the mobile speed, the data rate, the FEC code and the distance between the transmitter and the receiver. In this study, we gave an approximate expression for the probability of the number of retransmissions. We also presented simulation results which corroborated well with the approximation. Thus, we can use this technique to avoid lengthy simulations to obtain the end-to-end delay, and calculate the packet loss probability with different interleaving depths. Then for any given delay constraint and reliability requirement, we are able to do a table look-up to determine if an interleaver is preferred, and when needed, what the optimum interleaver size is.

References