Chapter 15

WIRELESS CHANNEL MODELS- COPING WITH COMPLEXITY

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Abstract In this work we explore two techniques to capture the behavior of wireless channels with mathematically tractable models. The first technique involves state-space aggregation to reduce a large number of states of a Markov chain to a fewer number of states. The property of strong and weak lumpability is discussed. The second technique involves stochastic bounding. These techniques are applied to three different previously published wireless channel models: mobile VHF, wireless indoor, and Rayleigh fading channels. Results show that our stochastic bounding technique can produce simple yet useful upper bounds for the original channel model. We investigate the goodness of these bounds through the performance of higher-layer error control protocols such as stop-and-go and TCP.

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1. INTRODUCTION

Errors occur on the wireless channel due to a variety of transmission impairments include fading, mobility, shadowing, interference and noise. Some of these impairments exhibit a significant degree of correlation. Markov models have been widely used in modeling communication channels to characterize this bursty behavior. The quest to develop models that adequately represent real channel behavior and that are mathematically tractable took several directions. The broad range of attempts include an initial two-state Markov model, some modifications to it, models with large number of states, those with restrictions on transitions, a general study on finite-state Markov chains, a subsequent study of infinite state models, higher order Markov models, and other variations [1].

The first attempt at channel modeling was done by Gilbert [2]. Although more general than the i.i.d. model, the two-state channel model is known to be inadequate for the representation of some time-varying channels. One way to overcome this problem is to enlarge the number of states. Fritchman [3] investigated a finite-state Markov chain model with $N$ ($N > 2$) states. The state space was then partitioned into two groups with Group A corresponding to $k$ error-free states and Group B corresponding to $N - k$ error states. Fritchman’s model has been applied by many researchers to represent error sequences obtained over the fading channels. Although a large number of states provides a better representation of the channel, the complexity of the model makes subsequent performance analysis intractable. It is therefore worth investigating if we can simplify the representation by reducing the $N$-state ($N > 2$) Markov chain to a two-state Markov chain and at the same time maintain a better representation of the channel. We shall outline two approaches in this regard, involving the notions of lumpability and hazard rate ordering.

The rest of the paper is organized as follows. In Section II, we briefly summarize the Markov characterization of three different types of digital fading channels that we selected for this study. In particular, we will consider the mobile VHF channel model developed by Swarts [4], wireless indoor channel model by Sivaprasakam [5], and the Rayleigh fading channel by Wang [6]. Section III describes techniques for reducing an $N$-state ($N > 2$) Markov chain to a two-state Markov chain. In Section IV, we determine if the state space of the three described channel models can be lumped into a two-state Markov chain without any loss of information. When this procedure fails, the channel is stochastically lower and upper bounded by two two-state Markov chains. The goodness of
the bounds is assessed by comparing the fraction of slots that the original chain spent in the good and bad states to those of the aggregated chains. We also study the goodness of the bounding process based on higher-layer error control protocols such as stop-and-go and TCP.

2. WIRELESS CHANNEL MODELS

In this section we briefly describe three previously published channel models. These channel models will be reexamined in section 4. with a view to simplifying their structure.

2.1 MOBILE VHF CHANNEL MODEL

The first model that we will consider for our state-space aggregation study is the digital fading mobile VHF channels developed by Swarts and Ferreira [4]. The model is based on a simplified Fritchman model, where there are $N-1$ good states and 1 bad state. The model was developed for studying the transmission of data from a motor vehicle traveling through an urban area. Four different modulation schemes, FSK, DPSK, QPSK, and 8-ary PSK, were used to transmit the data, and at the fixed receiving end, the error patterns that occurred were recorded. From these recordings, curves were fitted to the error-free runs to find the parameters of the partitioned Markov chain. In the model setup, the average vehicle speed for city center driving was 40 km/h. An FM transmitter was used to transmit the output of the modems at an RF carrier frequency of 145.2 MHz. The data (baud) rates in bits/s for the four different modulation schemes are FSK: 300 (300), DPSK: 1200 (1200), QPSK: 2400 (1200), and 8-ary PSK: 4800 (1600). The probability transition matrix for the four-state Fritchman channel model for different modulation schemes is shown in Table 15.1.

2.2 WIRELESS INDOOR CHANNEL MODEL

Another channel model which we study is a wireless indoor channel model developed by Sivaprakasam in [5]. Sivaprakasam used a forward-only recursion based hidden Markov modeling (FORM-HMM) technique for modeling burst errors in indoor digital channels. In [5], the Signal Processing Worksystem (SPW) was used to build an end-to-end-communication system to simulate the effects of channel in order to obtain the FORM-HMM models. The simulation measured errors in a QPSK modulation system with a bit rate of 10 kbits/s and sampling frequency of 40 kHz. The number of states in the HMM was six with three good and three bad states. Other simulation parameters include 30dB SNR. The transceivers were separated by 25 meters. The carrier
frequency is 1.28 GHz and the mobile unit speed is 1 m/s. The probability transition matrix for the six-state model of the indoor wireless channel is shown in Table 15.2.

### 2.3 Rayleigh Fading Channel Model

The two-state Gilbert-Elliot Markov model is not adequate for characterizing the dramatic and continuous channel quality changes in the Rayleigh fading channel. A Markov channel model with more than two states has to be formed to better represent the fading channel. In the work by Wang and Moayeri [6], a finite-state Markov channel (FSMC) was built by partitioning the received instantaneous signal-to-noise ratio (SNR) into $K$ intervals. The channel is represented by $K$-state stationary Markov chain with state space of $S = s_1, s_2, \ldots, s_K$. The state space $S$ is the set of $K$ different channel states with corresponding bit-error-rate $\epsilon_i$, $i \in \{1, 2, \ldots, K\}$. Let $S_n$ be the state at time $n$, $n = 0, 1, 2, \ldots$. The transition probability $P(i, j)$ and the steady state probability $\pi_i$ are

\[
P(i, j) = P[S_{n+1} = s_j | S_n = s_i], \quad i, j \in \{1, 2, \ldots, K\},
\]

\[
\pi_i = P[S_n = s_i], \quad i = 1, 2, \ldots, K,
\]

and

\[
P(k) = \sum_{i=1}^{K} P(k, i), \quad k = 1, 2, \ldots, K.
\]

The model allows the transitions only between adjacent states,

\[
P(k, i) = 0, \quad \text{if } |k - i| > 1.
\]

In a typical multipath propagation environment, the received signal envelope $r$ has the Rayleigh probability density function given by

\[
p(r) = \frac{r}{\sigma_r^2} \exp(-\frac{r^2}{2\sigma_r^2}), \quad r \geq 0,
\]

where $\sigma_r^2$ is the variance of the in-phase and quadrature components of the received signal. With additive Gaussian noise of variance $\sigma_0^2$, the received instantaneous SNR $\gamma = \frac{r^2}{2\sigma_0^2}$ is distributed exponentially with probability density function

\[
p(\gamma) = \frac{1}{\gamma_0} \exp(-\frac{\gamma}{\gamma_0}), \quad \gamma \geq 0,
\]

where $\gamma_0 = \frac{\sigma_r^2}{\sigma_0^2}$ is the average SNR. The fading characteristics of the signal envelope are determined by the Doppler frequency. The level
crossing rate of the instantaneous SNR process $\gamma$ is the average number of times per unit interval that a fading signal crosses a given signal level $\Gamma$. For a random distribution of direction of motion providing a maximum Doppler frequency $f_m$, the level crossing rate of level for the SNR process is

$$N(\Gamma) = \sqrt{\frac{2\pi \Gamma}{\gamma_0}} f_m \exp\left(-\frac{\Gamma}{\gamma_0}\right).$$

(15.7)

A finite-state Markov channel model can be built to represent the time-varying behavior of the Rayleigh fading channel. Let $\Gamma = \{\Gamma_1, \Gamma_2, \ldots, \Gamma_{K+1}\}$ be the received SNR thresholds in increasing order with $\Gamma_1 = 0$ and $\Gamma_{K+1} = \infty$. The channel is in state $k$ if the SNR is between $\Gamma_k$ and $\Gamma_{k+1}$. For our study, we consider an example from the paper [6]. In this model, an eight-state Markov channel model was used with the maximum Doppler frequency $f_m$ equals to 10 Hz. The transmission was $10^5$ symbols/s and the SNR thresholds are chosen such that the steady state probability of being in any state is equally likely. Table 15.3 shows the transition probability $P(i, j)$ from Wang’s analytical model.

3. STATE-SPACE AGGREGATION TECHNIQUES

In this section, we describe two broad techniques for reducing the complexity of channel models. The first technique for aggregation is applicable to Markov chains while the second technique for bounding is applicable to a larger class of processes.

3.1 LUMPABLE CHAINS

Let $X$ be a homogeneous, irreducible, discrete-time Markov chain on a finite-state space denoted by $S$, which without loss of generality we assume to be a subset of the natural numbers $N$, $S = \{1, 2, \ldots, N\}$. The process $X$ has an aperiodic transition matrix $P$ with an equilibrium distribution denoted by a row vector $\pi$ and initial probability vector $A$ where $A$ is the set of all probability vectors. Let $\Omega = \{\Omega(1), \Omega(2), \ldots, \Omega(M)\}$ with $M < N$ be a fixed partition of the state space $S$.

With the given process $X$ and the partition $\Omega$, we can associate an aggregated stochastic process $Y$ with values on $S = \{1, 2, \ldots, M\}$, defined by $Y_n = m \iff X_n \in \Omega(m), \forall n \geq 0, m = 1, 2, \ldots, M$. Conditions under which the aggregated stochastic process $agg(\alpha, P, \Omega)$ is a homogeneous Markov chain $\forall \alpha \in A$ were studied by Burke and Rosenblatt in 1958 [8].
Hachgian in 1963 [9], and Kemeny and Snell in 1967 [10]. A chain that has this property is called strongly lumpable with respect to the partition $\Omega$. A more general problem is to determine if there exists some initial distributions $\alpha$ such that $agg(\alpha, P, \Omega)$ is a homogeneous Markov chain but not necessarily for every vector in $A$. The Markov chain, in this case, is called weakly lumpable with respect to the partition $\Omega$.

Weakly lumpability was first studied by Kemeny and Snell in 1976 [10], where they showed that it is possible that there exists a proper subset $A_M$ of the set of all initial probability vectors in $A$ such that the aggregated process $Y$ is a homogeneous Markov process if and only if $\alpha \in A_M$. Kemeny and Snell also provided a simple but strong sufficient condition for weak lumpability. Rubino and Sericola in 1991 [12] obtained a characterization of weak lumpability by means of an algorithm which computes the set $\hat{A}_M$ of initial distributions and gave necessary and sufficient conditions for weak lumpability. Lumpability of a Markov chain with a denumerable state space was first discussed by Hachgian in 1963 [9]. Rubino and Sericola in [12] gave an algorithm to compute the set $\hat{A}_M$.

In the following sections, we will present the algorithm for determining the existence of the proper subset $\hat{A}_M$ and give some examples to illustrate the notions of strong and weak lumpability. In our application, the initial distribution vector is always the steady-state distribution vector, i.e. $\alpha = \pi$. Hence, proving that there exists a proper subset $\hat{A}_M$ is sufficient, since $\hat{A}_M \neq \emptyset \Rightarrow \pi \in \hat{A}_M$.

### 3.2 ALGORITHM TO DETERMINE EXISTENCE OF $A_M$

Before presenting the algorithm for determining the existence of the set $\hat{A}_M$, we need the following notation. Let $\pi^{\Omega(l)}$ be obtained from the steady-state distribution, $\pi$, by setting to zero all entries corresponding to states which are not in $\Omega(l)$. We define

$$\hat{P}(l, m) = \sum_{i \in \Omega(l)} \pi^{\Omega(l)}(i)P(i, \Omega(m)) \quad l, m \in S,$$

and $\hat{P}_l$ be the $l$th row of the transition matrix $\hat{P}$, where $\hat{P}$ is the probability transition matrix of the aggregated chain if the original chain is lumpable. For each $l \in \hat{S}$, we define the following matrices

$$\hat{P}_l = (P(i, \Omega(k)) \quad i \in \Omega(l), k \in \hat{S},$$

$$H_l = \hat{P}_l - 1^T \hat{P}_l,$$
with 1 as a row vector where the dimension is defined by context and $T$ denotes the transposition. Let $P_l$ be the submatrix of $P$ constituted by the transition probabilities from the states of $\Omega(l)$ to the states of $S$,

$$P_l = (P(\Omega(l), \Omega(0)) \ldots P(\Omega(l), \Omega(k)) \ldots).$$

We define the block matrices

$$H^{[1]} = H$$
$$H^{[j+1]} = \text{Diag}(P_l H^{[j]}), \quad j \geq 1,$$

and the convex sets, for all $j \geq 1$, $\mathcal{A}^j = \{\alpha \in \mathcal{A} | \alpha H^{[k]} = 0, \text{for } 1 \leq k \leq j\}$, where $\mathcal{A}^j$ is known as a polytope of $\mathbb{R}^N$ [14].

To determine the existence of the set $\mathcal{A}_M$, we can proceed as follows. If the chain is not strongly lumpable with respect to the partition $\Omega$, then $\mathcal{A}_M = \mathcal{A}$. If not, we first verify whether for all $l \in \hat{S}$ the vector $\pi^{\Omega(l)} P$ is in $\mathcal{A}^1$. If there exists $l \in \hat{S}$ such that $\pi^{\Omega(l)} P \notin \mathcal{A}^1$, then $\pi \notin \mathcal{A}^2$ and $\mathcal{A}_M = \emptyset$. The loop terminates if the condition of $\mathcal{A}_M = \emptyset$ is encountered. Otherwise, the loop is continued for $N$ iterations. Note that verifying $\mathcal{A}_M \neq \emptyset$ at each stage is equivalent to checking $\pi^{\Omega(l)} P H^{[j]} = 0$, for $l \in \hat{S}$ and $j = 1, 2, \ldots, N$. See [12] for additional details. Hence, the algorithm can be expressed as follows:

```python
if X is strongly lumpable then $\mathcal{A}_M := \mathcal{A}$
    stop
else
    $\mathcal{A}_M := \emptyset$
    for $j = 1$ to $N$
        if $\exists l \in \hat{S} : \pi^{\Omega(l)} P H^{[j]} \neq 0$
            empty := true
            $\mathcal{A}_M := \emptyset$
            break
        endif
    endfor
endif
```

### 3.3 EXAMPLES OF LUMPABLE CHAINS

In this section, we provide some numerical examples to illustrate the concept of strong and weak lumpability.

**Example 1**-Strong Lumpability [10]
Let the probability transition matrix \( P \) be
\[
P = \begin{pmatrix}
\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & 0
\end{pmatrix}
\]
(15.13)
and the fixed partition \( \Omega = \{\Omega(1), \Omega(2)\} \), \( \Omega(1) = \{1, 2\} \) and \( \Omega(2) = \{3\} \). We note that the probability of moving from either of states 1 or 2 to state 3 is the same, i.e. \( P(1,3) = P(2,3) = \frac{1}{4} \). Hence, the strong lumpability condition is satisfied with the given partition \( \Omega \) The probability transition matrix of the aggregated chain is
\[
\hat{P} = \begin{pmatrix}
\frac{3}{4} & \frac{1}{4} \\
1 & 0
\end{pmatrix}
\]
(15.14)
Note that the condition of strong lumpability is not satisfied for the partition \( B = \{B(1), B(2)\} \) with \( B(1) = \{1\} \) and \( B(2) = \{2,3\} \), since \( P(2, B(1)) = P(2, 1) = \frac{1}{4} \) and \( P(3, B(1)) = P(3, 1) = \frac{1}{2} \).

**Example 2-Weak Lumpability [11]**
Let the probability transition matrix \( P \) be
\[
P = \begin{pmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\
0 & \frac{1}{6} & \frac{5}{6} \\
\frac{7}{8} & \frac{1}{8} & 0
\end{pmatrix}
\]
(15.15)
and \( \Omega = \{\Omega(1), \Omega(2)\} \), \( \Omega(1) = \{1\} \) and \( \Omega(2) = \{2,3\} \). Applying the algorithm in the previous section, it can be shown that \( A_M \) exists, where \( A_M = \{\lambda(1,0,0) + (1 - \lambda)(0,1/3,2/3), 0 \leq \lambda \leq 1\} \).

### 3.4 STOCHASTIC BOUNDS

When a chain fails to be strongly or weakly lumpable, an alternative approach is to stochastically bound (from above and below) the given chain with geometrically distributed on-off processes. It is of course possible that in some instances no such bounding processes may exist. Fortunately, in [15] Sonderman provides sufficient conditions for the existence of such bounding processes. Before presenting Sonderman’s result some definitions on stochastic ordering are necessary.

Let \( F_x, f_x, F_y \) and \( f_y \) be the cumulative distribution and density functions of two renewal processes \( X \) and \( Y \) respectively. The process \( X \) is said to be greater than the process \( Y \) in the sense of hazard rate ordering if and only if \( \forall x, \frac{f_Y(x)}{1-F_Y(x)} = r_Y(x) > r_X(x) = \frac{f_X(x)}{1-F_X(x)} \). Consider now two
On-Off processes $X^1$ and $X^2$ characterized by the absolutely continuous cumulative distribution functions $F_1, G_1, F_2,$ and $G_2$ with conditional failure rate functions $r_1, s_1, r_2, s_2$ respectively and initial conditions $p^1$ and $p^2$. Sonderman in [15] showed that if $p^1 \leq p^2$ and $r_1(u) \leq r_2(v)$ and $s_1(u) \leq s_2(v), \forall u, v,$ then there exist two semi-Markov processes $\hat{X}^1$ and $\hat{X}^2$ on the same probability space such that $\mathcal{L}(\hat{X}^i) = \mathcal{L}(X^i)$ for $i = 1, 2$ and $P[\hat{X}^1(t) \leq \hat{X}^2(t), \forall t \geq 0] = 1$.

To use this result, we would like to set the hazard rates of the bounding processes to be a constant. This corresponds to an exponential or geometric distribution for the bounding on and off periods. If it turns out that the hazard rates of the on and off periods of the actual process are bounded away from zero and infinity, then the upper and lower bounds to the hazard rate will specify the parameters of the geometrically distributed (a.s.) bounding On-Off processes. So the task reduces to (1) ensuring that the actual process is indeed an on-off process and (2) computing the sojourn time probability distributions in the aggregated states. The hazard rates are then computed for the sojourn time probability distributions and the max and min values of the hazard rate curves are identified.

In general, aggregated process may not always evolve as an On-Off process even if the underlying chain is Markovian. Nonetheless, it can be shown that if either one of the two aggregated states contains only a single state or if there exists a single gateway (state) to and from the two aggregated states then the evolution of the aggregated process satisfies the requirements of an On-Off process. A chain is said to have a gateway state if all transitions from the good aggregated state to the bad aggregated state must pass through the singleton good gateway state and similarly for transitions from the bad aggregated states to the good.

The proof of this result requires us to establish that good and bad dwell periods are IID. The proof, which is omitted for brevity, follows from conditioning the underlying chain on the singleton gateway states and exploiting the renewal nature of the subsequent evolution.

If these conditions do not exist in the original Markov chain, we develop a looser bounding technique by selective reassignment of the states in each of aggregated sets and finding those state-space partitions that would provide the tightest bounds from above and below of the original process.

**Computation of Sojourn Times.** Without loss of generality, let $\hat{S} = \{1, 2\}$ and the partition $\Omega = \{\Omega(1), \Omega(2)\}$ of the state space. $S$ induces a decomposition of probability transition matrix $P$ into four
submatrices and a decomposition of steady state probability distribution vector $\pi$ into two subvectors:

$$P = \begin{pmatrix}
  P_{\Omega(1)\Omega(1)} & P_{\Omega(1)\Omega(2)} \\
  P_{\Omega(2)\Omega(1)} & P_{\Omega(2)\Omega(2)}
\end{pmatrix} \quad (15.16)$$

and

$$\pi = (\pi_{\Omega(1)}^{\Omega(1)} \pi_{\Omega(2)}^{\Omega(2)}). \quad (15.17)$$

We define the sojourn of stochastic process $X$ in $\Omega(1)$ as any sequence $X_m, X_{m+1}, \ldots, X_{m+k}$ where $k \geq 1$, $X_m, X_{m+1}, \ldots, X_{m+k-1} \in \Omega(1)$, $X_{m+k} \notin \Omega(1)$ and if $m > 0$, $X_{m-1} \notin \Omega(1)$. This sojourn begins at time $m$ and finishes at time $m + k$. It lasts $k$ steps. We denote $N_{\Omega(1)}$ as a random variable taking values in $\mathcal{N}$ to represent the sojourn time of the process $X$ in $\Omega(1)$. In [13], the following explicit expression of the sojourn time distribution of $N_{\Omega(1)}$ can be obtained. For any $k \geq 1$, the sojourn time distribution of $N_{\Omega(1)}$ is

$$P(N_{\Omega(1)} = k) = v P_{\Omega(1)\Omega(1)}^{k-1} (I - P_{\Omega(1)\Omega(1)})^T$$

for any $k \geq 1$, \quad (15.18)

where $v$ is a vector given by: $v = (1/K) \pi_{\Omega(1)}^{\Omega(1)} (I - P_{\Omega(1)\Omega(1)})$ with $K$ as a normalization constant equal to $\pi_{\Omega(1)}^{\Omega(1)} (I - P_{\Omega(1)\Omega(1)})^T$, is the identity matrix. The sojourn time distribution, $P(N_{\Omega(2)} = k)$, can be obtained in a similar manner by replacing $\Omega(1)$ with $\Omega(2)$.

4. NUMERICAL RESULTS

We apply the state-space aggregation techniques presented in the previous section to the three different wireless channel models to reduce an $N$-state ($N > 2$) Markov chain to a two-state Markov chain. The two-state aggregated chain composes of a good state $G$ and a bad or burst state $B$ and with the following probability transition matrix

$$P = \begin{pmatrix}
  p_{gg} & p_{gb} \\
  p_{bg} & p_{bb}
\end{pmatrix}, \quad (15.19)$$

where $p_{gb} = 1 - p_{gg} = P[G \rightarrow B]$ and $p_{bg} = 1 - p_{bb} = P[B \rightarrow G]$.

4.1 PERFORMANCE OF BOUNDS

Our analysis indicated that none of the channel models is strongly or weakly lumpable with the specified partition. However, the model developed by Wang for Rayleigh channel is nearly weakly lumpable. We then applied the stochastic bounding technique to all channel models. To proceed with the stochastic bounding technique, we first compute the
sojourn time probability distribution. It can be easily shown that the
sojourn probability distribution $P[N = k]$ and its corresponding hazard
rate $\beta(k)$ in the aggregated state have the following forms:

$$P[N = k] = c_1 a_1^k + c_2 a_2^k + c_3 a_3^k \quad (15.20)$$

and

$$\beta(k) = \frac{c_1 a_1^k + c_2 a_2^k + c_3 a_3^k}{c_1 a_1^{\frac{1}{1-a_1}} + c_2 a_2^{\frac{1}{1-a_2}} + c_3 a_3^{\frac{1}{1-a_3}}} \quad (15.21)$$

The coefficients $c_i$ and $a_i$ are constants. Note that for the VHF channel
model, the sojourn time probability distribution of the aggregated 
bad state is geometrically distributed since the aggregated bad state con-
tains only a single state of the original chain. Our analysis indicated that
the hazard rates of all the considered channel models are bounded away
from 0 and $\infty$. Hence, the original channel models can be stochastically
upper and lower bounded by two simpler Markov processes.

Fig. 15.1 shows the hazard rate as a function of time slot $k$ for ag-
aggeregated good states for the mobile VHF channel model for the city
environment. The hazard rate of the aggregated good state has the
maximum and minimum values of 0.382 and 0.001 for all values of $k$,
respectively. Note that these values correspond to $1 - p_{gg}$. The aggre-
gated bad state contains a single state of the original chain. The optimi-
tic (upper bound) estimation of the channel can be formed by taking
the minimum value of the hazard rate function of the aggregated good
state. Similarly, the pessimistic (lower bound) estimation of the channel
is formed by taking the maximum value of the hazard rate function of
the aggregated good state. Table 15.4 shows the transition probabilities
for both bounds for all channel models.

To cross check the optimistic and pessimistic channel models derived
above, we verify if the actual channel model falls between these two
channel bounds through simulation as well. The numerical values Table
15.5 indicate that the stochastic bound of the optimistic channel model
is very tight since the fractions of good and bad slots are very close to
the actual channel model while the lower stochastic bound is looser.

We had to apply the gateway-state stochastic bound technique to the
wireless indoor channel model because neither the good nor the bad
states are singletons or there exists a gateway state going one aggre-
gated state to another. The original process has the following fixed
partition $G = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. We create the following new
partitions for the aggregated good and bad state respectively: $G_a^1 = \{1\}, G_a^2 = \{2\}, G_a^3 = \{3\}, B_a^1 = \{2, 3, 4, 5, 6\}, B_a^2 = \{1, 3, 4, 5, 6\}$ and
The resulting processes $X_a^1$, $X_a^3$, and $X_a^2$ corresponding to these partitions are alternating renewal processes which bound the original process from below. Similarly, we create the following partitions: $G_b^1 = \{1\}, G_b^2 = \{2\}, G_b^3 = \{3\}, B_b^1 = \{2, 3, 4, 5, 6\}, B_b^2 = \{1, 3, 4, 5, 6\}$ and $B_b^3 = \{1, 2, 4, 5, 6\}$. The resulting processes $X_b^1$, $X_b^2$, and $X_b^3$ corresponding to these partitions are alternating renewal processes which bound the original process from above. Table 15.6 shows the numerical result when the gateway-state stochastic bounding technique is applied to the wireless indoor channel model. Note that the tightest upper bound to the original process has the state-space partition of $G = \{1, 2, 3, 4, 6\}$ and $B = \{5\}$, where the tightest lower bound has the following partition: $G = \{1\}$ and $B = \{2, 3, 4, 5, 6\}$.

4.2 GOODNESS OF BOUNDS FROM HIGHER-LAYER PERSPECTIVE

The issue of the quality of the channel bounds can be addressed at different levels. One could develop a measure of the goodness at the physical layer. But, given our interest in data transmission, we have explored via simulations the goodness of the bounding process as seen by stop-and-go ARQ protocol and Transmission and Control Protocol (TCP). In both cases we make the assumption that the channel error models are applicable at the packet level. In principle one could take the bit level error models and extract a packet level error model based on further details of the system. We have not attempted to do so at this time but see [16] for additional details on this aspect.

For the stop-and-go simulation model, the source attempts to transmit one packet at a time to the destination. If the source receives an acknowledgment of the transmitted packet, the next packet in the queue will be transmitted. We assume that the source has an infinite number of packets to transmit and the retransmission timeout is set to be five times the round-trip transmission/propagation delay. Table 15.7 shows the comparison of the stop-and-go protocol performance for different channel models. We can see that the bounds are preserved. We also observed that the performance of the stop-and-go protocol over the optimistic channel model is very close to that of the actual model.

Table 15.8 shows the performance of TCP for the three different channel models. In our TCP simulation model, we assume the following TCP parameters: deviation gain, 0.25; initial Retransmission Timeout (RTO), 1.0 sec.; Karns algorithm, enabled; maximum RTO, 240 sees; minimum RTO, 0.5 sec.; maximum acknowledgment delay, 0.001; Nagel silly window syndrome, enabled; persistence timeout, 1.0 sec; Round Trip Time
(RTT) deviation coefficient, 4.0; RTT gain, 0.125; and receiver buffer capacity, 65 KBytes. The performance of TCP degrades significantly as the channel has longer burst period. This is resulted from both retransmission and TCP’s reduction of its congestion window. We also observe that the bounds are still preserved. However, the upper bound is not as tight as observed in the stop-and-go protocol performance.

5. CONCLUSIONS

In this paper, we studied techniques for developing tractable channel models. The first step was to identify accurate channel models. A number of published results that involved large order Markov channel models were identified. The next step was to identify conditions under which the models could be simplified. We showed that if the Markov chain is either strongly or weakly lumpable relative to a particular partition, the evolution of the original channel model can be exactly described by a 2-state Markov chain. When the channel model fails to be strongly or weakly lumpable, a hazard rate ordering technique can be applied to stochastically upper and lower bound the original channel model with a 2-state model. We also study the goodness of the bounding process based on higher-layer error control protocols, stop-and-go and TCP. We found that the bounds were preserved and that in both instances the upper bound was very tight.

Figure 15.1 Hazard rate function for aggregated GOOD state for VHF channel model using DPSK modulation scheme for the city environment.
Table 15.1  Transition probabilities for the four-state Fritchman model for the city environment using FSK, DPSK, QPSK, and 8-ary PSK modulation scheme [4].

<table>
<thead>
<tr>
<th>Prob.</th>
<th>FSK</th>
<th>DPSK</th>
<th>QPSK</th>
<th>PSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(1,1)</td>
<td>0.515789</td>
<td>0.467321</td>
<td>0.530715</td>
<td>0.594808</td>
</tr>
<tr>
<td>P(2,2)</td>
<td>0.995066</td>
<td>0.900030</td>
<td>0.999489</td>
<td>0.999573</td>
</tr>
<tr>
<td>P(3,3)</td>
<td>0.999624</td>
<td>0.999464</td>
<td>0.999950</td>
<td>0.999961</td>
</tr>
<tr>
<td>P(4,4)</td>
<td>0.453882</td>
<td>0.385774</td>
<td>0.466127</td>
<td>0.439773</td>
</tr>
<tr>
<td>P(4,1)</td>
<td>0.437937</td>
<td>0.416705</td>
<td>0.480427</td>
<td>0.493091</td>
</tr>
<tr>
<td>P(4,2)</td>
<td>0.070908</td>
<td>0.124011</td>
<td>0.041837</td>
<td>0.055990</td>
</tr>
<tr>
<td>P(4,3)</td>
<td>0.037272</td>
<td>0.073510</td>
<td>0.011609</td>
<td>0.011146</td>
</tr>
<tr>
<td>P(1,4)</td>
<td>0.484211</td>
<td>0.532679</td>
<td>0.469284</td>
<td>0.405192</td>
</tr>
<tr>
<td>P(2,4)</td>
<td>0.004934</td>
<td>0.099970</td>
<td>0.000511</td>
<td>0.000427</td>
</tr>
<tr>
<td>P(3,4)</td>
<td>0.000376</td>
<td>0.000536</td>
<td>0.000050</td>
<td>0.000039</td>
</tr>
</tbody>
</table>

Table 15.2  Transition probabilities for six-state indoor wireless channel model [5].

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.951</td>
<td>0.000</td>
<td>0.000</td>
<td>0.047</td>
<td>0.003</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>0.695</td>
<td>0.100</td>
<td>0.023</td>
<td>0.182</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.001</td>
<td>0.514</td>
<td>0.485</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.077</td>
<td>0.760</td>
<td>0.112</td>
<td>0.000</td>
<td>0.052</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.045</td>
<td>0.717</td>
<td>0.000</td>
<td>0.000</td>
<td>0.238</td>
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</tbody>
</table>

Table 15.3  Transition probabilities for eight-state Markov channel with the maximum Doppler frequency $f_m = 10Hz$ [6].

<table>
<thead>
<tr>
<th>k</th>
<th>P(k,k-1)</th>
<th>P(k,k)</th>
<th>P(k,k+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k = 1</td>
<td>-</td>
<td>0.999359</td>
<td>0.000641</td>
</tr>
<tr>
<td>k = 2</td>
<td>0.000641</td>
<td>0.998552</td>
<td>0.000807</td>
</tr>
<tr>
<td>k = 3</td>
<td>0.000807</td>
<td>0.998334</td>
<td>0.000859</td>
</tr>
<tr>
<td>k = 4</td>
<td>0.000859</td>
<td>0.998306</td>
<td>0.000835</td>
</tr>
<tr>
<td>k = 5</td>
<td>0.000835</td>
<td>0.998420</td>
<td>0.000745</td>
</tr>
<tr>
<td>k = 6</td>
<td>0.000745</td>
<td>0.998665</td>
<td>0.000590</td>
</tr>
<tr>
<td>k = 7</td>
<td>0.000590</td>
<td>0.999048</td>
<td>0.000361</td>
</tr>
<tr>
<td>k = 8</td>
<td>0.000361</td>
<td>0.999639</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 15.4  Stochastic bounds of hazard rates for different channel models.

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th></th>
<th>Pessimistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_{gg}$</td>
<td>$p_{bb}$</td>
<td>$p_{gg}$</td>
<td>$p_{bb}$</td>
</tr>
<tr>
<td>FSK</td>
<td>0.999624</td>
<td>0.453882</td>
<td>0.611000</td>
<td>0.453882</td>
</tr>
<tr>
<td>DPSK</td>
<td>0.999464</td>
<td>0.385774</td>
<td>0.697412</td>
<td>0.385774</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.999950</td>
<td>0.466127</td>
<td>0.577654</td>
<td>0.466127</td>
</tr>
<tr>
<td>PSK</td>
<td>0.999961</td>
<td>0.439773</td>
<td>0.643322</td>
<td>0.439773</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>0.999901</td>
<td>0.997249</td>
<td>0.999168</td>
<td>0.999686</td>
</tr>
</tbody>
</table>

Table 15.5  Comparison of optimistic and pessimistic channel models with the actual channel models using hazard rate ordering technique.

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th></th>
<th>Actual</th>
<th></th>
<th>Pessimistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% good</td>
<td>% bad</td>
<td>% good</td>
<td>% bad</td>
<td>% good</td>
<td>% bad</td>
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<tr>
<td>FSK</td>
<td>0.99938</td>
<td>6.88e-4</td>
<td>0.9913</td>
<td>0.0087</td>
<td>0.5840</td>
<td>0.4160</td>
</tr>
<tr>
<td>DPSK</td>
<td>0.99911</td>
<td>8.71e-4</td>
<td>0.9929</td>
<td>0.0071</td>
<td>0.6700</td>
<td>0.3300</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.99993</td>
<td>9.36e-5</td>
<td>0.9970</td>
<td>0.0030</td>
<td>0.5583</td>
<td>0.4417</td>
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<tr>
<td>PSK</td>
<td>0.99999</td>
<td>6.96e-5</td>
<td>0.9976</td>
<td>0.0024</td>
<td>0.6110</td>
<td>0.3890</td>
</tr>
<tr>
<td>Indoor</td>
<td>0.9987</td>
<td>0.0013</td>
<td>0.9843</td>
<td>0.0157</td>
<td>0.7219</td>
<td>0.2781</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>0.9653</td>
<td>0.0347</td>
<td>0.9168</td>
<td>0.0757</td>
<td>0.2740</td>
<td>0.7260</td>
</tr>
</tbody>
</table>

Table 15.6  Comparison of optimistic and pessimistic channel models with the actual channel models using an exit-gateway bounding technique.

<table>
<thead>
<tr>
<th>State Decomposition</th>
<th>Percentage of Good Slots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
</tr>
<tr>
<td>{1, 2, 3, 4, 5}</td>
<td>{6}</td>
</tr>
<tr>
<td>{1, 2, 3, 4, 6}</td>
<td>{5}</td>
</tr>
<tr>
<td>{1, 2, 3, 5}</td>
<td>{4}</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td>{4, 5, 6}</td>
</tr>
<tr>
<td>{1}</td>
<td>{2, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>{2}</td>
<td>{1, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>{3}</td>
<td>{1, 2, 4, 5, 6}</td>
</tr>
</tbody>
</table>
Table 15.7 Comparison of Stop and Go protocol performance for optimistic, pessimistic, and actual channel models ($d =$ delay, $\gamma =$ throughput).

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th></th>
<th>Actual</th>
<th></th>
<th>Pessimistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d$</td>
<td>$\gamma$</td>
<td>$d$</td>
<td>$\gamma$</td>
<td>$d$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>FSK</td>
<td>1.0009</td>
<td>0.9991</td>
<td>1.0092</td>
<td>0.9909</td>
<td>1.7125</td>
<td>0.5839</td>
</tr>
<tr>
<td>DPSK</td>
<td>1.0011</td>
<td>0.9988</td>
<td>1.0071</td>
<td>0.9930</td>
<td>1.4925</td>
<td>0.6700</td>
</tr>
<tr>
<td>QPSK</td>
<td>1.0003</td>
<td>0.9996</td>
<td>1.0016</td>
<td>0.9984</td>
<td>1.7921</td>
<td>0.5580</td>
</tr>
<tr>
<td>PSK</td>
<td>1.0003</td>
<td>0.9997</td>
<td>1.0021</td>
<td>0.9980</td>
<td>1.6359</td>
<td>0.6113</td>
</tr>
<tr>
<td>Indoor</td>
<td>1.0014</td>
<td>0.9986</td>
<td>1.0091</td>
<td>0.9908</td>
<td>1.3851</td>
<td>0.7220</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>1.0134</td>
<td>0.9877</td>
<td>1.0000</td>
<td>0.9009</td>
<td>1.4599</td>
<td>0.7654</td>
</tr>
</tbody>
</table>

Table 15.8 Comparison of TCP performance for optimistic, pessimistic, and actual channel models ($d =$ delay, $\gamma =$ throughput).

<table>
<thead>
<tr>
<th></th>
<th>Optimistic</th>
<th></th>
<th>Actual</th>
<th></th>
<th>Pessimistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d$</td>
<td>$\gamma$</td>
<td>$d$</td>
<td>$\gamma$</td>
<td>$d$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>FSK</td>
<td>0.3283</td>
<td>0.9999</td>
<td>24.20</td>
<td>0.9494</td>
<td>122.30</td>
<td>0.7959</td>
</tr>
<tr>
<td>DPSK</td>
<td>0.3294</td>
<td>0.9960</td>
<td>17.98</td>
<td>0.9959</td>
<td>77.33</td>
<td>0.7345</td>
</tr>
<tr>
<td>QPSK</td>
<td>0.3258</td>
<td>0.9991</td>
<td>50.33</td>
<td>0.9161</td>
<td>139.80</td>
<td>0.7578</td>
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<tr>
<td>PSK</td>
<td>0.3258</td>
<td>0.9999</td>
<td>29.25</td>
<td>0.9606</td>
<td>125.12</td>
<td>0.8114</td>
</tr>
<tr>
<td>Indoor</td>
<td>0.3276</td>
<td>0.9995</td>
<td>4.438</td>
<td>0.9888</td>
<td>15.84</td>
<td>0.9356</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>0.3227</td>
<td>0.6000</td>
<td>0.3209</td>
<td>0.4379</td>
<td>0.8977</td>
<td>0.2146</td>
</tr>
</tbody>
</table>
References


